TOPOLOGICAL MANIFOLDS || PROBLEM SHEET 10

Problem 1. Prove the "strong topological Palais theorem". That is, let $n \geq 6$, let M be a connected n-manifold, and let $\phi, \psi \colon D^n \to \operatorname{Int} M$ be locally collared embeddings. Then there is an isotopy $H_t \colon M \to M$ where each H_t is a homeomorphism, $H_0 = \mathrm{Id}$, and $H_1 \circ \psi = \phi$.

Hint: H_1 is the outcome of the weak version of the theorem from the previous problem set. Upgrade the proof of homogeneity of closed manifolds to produce an isotopy of the maps ψ and ϕ, then apply the isotopy extension theorem.

Problem 2. Let M be a compact manifold. Prove that $\mathrm{Homeo}(\mathrm{Int}(M))$ is locally contractible.

Recall that we saw earlier that the homeomorphism group of a noncompact manifold need not be locally contractible. The above gives an alternative proof that $\mathrm{Homeo}(\mathbb{R}^n)$ is locally contractible.

Hint: Let C be the compact manifold formed by removing an open collar of the boundary of M. Argue that a neighbourhood of the identity map in $\mathrm{Homeo}(\mathrm{Int}(M))$ can be deformed into $\mathrm{Homeo}_C(M)$, consisting of the homeomorphisms of M which restrict to the identity on C. Now deform $\mathrm{Homeo}_C(M)$ to $\{\mathrm{Id}\}$ using the collar.