TOPOLOGICAL MANIFOLDS || PROBLEM SHEET 11

Fix an orientation on S^m for every m. Let $f\colon S^n \to S^{n+2}$ be a locally flat embedding. We call $K := f(S^n)$ an n-knot. If S^n and S^{n+2} have their standard smooth structures, and if f is a smooth embedding, then we call K a smooth n-knot.

Problem 1. For $n \geq 5$, show that the embeddings f and g defining two n-knots $K = f(S^n)$ and $J = g(S^n)$ are locally-flat isotopic if and only if there is an orientation preserving homeomorphism $F\colon S^{n+2} \to S^{n+2}$ such that $F(K) = J$, and $F|_K\colon K \to J$ is orientation preserving, with respect to the orientations induced by f and g.

You may use the isotopy extension theorem, as well as SH_m and its consequences. (The same holds for all $n \geq 1$, but we do not have the tools to prove it from the course.)

Problem 2. For $n \geq 1$, show that the embeddings f and g defining two smooth n-knots $K = f(S^n)$ and $J = g(S^n)$ are smoothly isotopic if and only if there is an orientation preserving diffeomorphism $F\colon S^{n+2} \to S^{n+2}$ such that $F(K) = J$ and $g^{-1} \circ F \circ f\colon S^n \to S^n$ is smoothly isotopic to the identity.

You may use the smooth version of the isotopy extension theorem. The theorems from the course may not be very helpful.