TOPOLOGICAL MANIFOLDS || PROBLEM SHEET 2

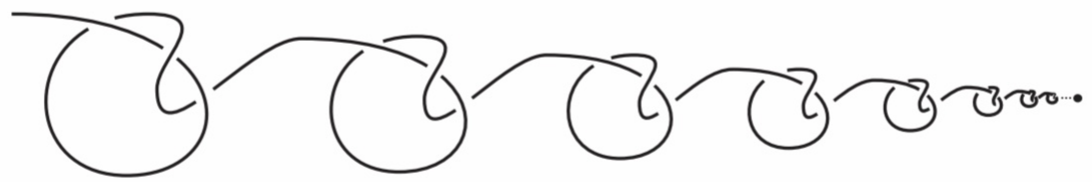

FIGURE 1. The arc γ

Problem 1. Prove that the arc γ in Figure 1 is locally flat, and indeed there is a homeomorphism f of pairs mapping $(\mathbb{R}^3, \gamma)$ to $(\mathbb{R}^3, [0,1])$.

Hint: Find a nested sequence of balls $\{B_i\}$ so that $\cap B_i$ is the compactification point and each B_i intersects γ at a single point. For each i there is an isotopy that is the identity on $(S^3 \setminus \operatorname{Int} B_i) \cup B_{i+1}$ and that straightens out $\gamma \cap (B_i \setminus \operatorname{Int} B_{i+1})$. The desired homeomorphism f is a limit of a composition of such homeomorphisms.

FIGURE 2. The arc δ

Problem 2. The arc δ is the union of γ and a standard interval $[0,1]$ (see Figure 2). Prove that δ is not locally flat.

Hint: use the Seifert-van Kampen theorem to prove that δ is not 1-alg at the "union point".

Problem 3. Let M be an n-dimensional manifold with nonempty boundary. Let U be an open subset of ∂M that is collared, that is there exists an embedding $U \times [0,1] \hookrightarrow M$ with $(u, 0) \mapsto u$ for all $u \in U$. Let $C \subseteq U$ be a closed subset. Then there exists a collaring of ∂M extending the given collaring on C.