TOPOLOGICAL MANIFOLDS || PROBLEM SHEET 3

Problem 1. Is the double Fox-Artin arc in the interior of D^3 cellular?

Problem 2. Let M be a compact n-manifold so that $M = U_1 \cup U_2$ where each U_i is homeomorphic to $\mathbb{R}^n$.

(a) Prove that M is homeomorphic to S^n. You may use the Schoenflies theorem.

(b) Conclude that if a closed n-manifold M is an (unreduced) suspension SX for some space X, then M is homeomorphic to the sphere S^n.

Note: (b) reduces the double suspension problem to showing that the double suspension is a manifold, not specifically a sphere.

Problem 3. Let $\Sigma \subseteq S^n$ be an embedded copy of S^{n-1} and let U be one of the two path components of $S^n \setminus \Sigma$. If the closure $\overline{U}$ is a manifold, then $\overline{U}$ is homeomorphic to D^n.

Problem 4. Let $f\colon D^n \to D^n$ be a locally collared embedding of a disc into the interior of a disc. Prove that $D^n \setminus f(D^n)$ is homeomorphic to $S^{n-1} \times (0,1]$. Hint: show that $f(D^n)$ is cellular.

Note, the result that $D^n \setminus \mathrm{Int}(f(D^n))$ is homeomorphic to $S^{n-1} \times [0,1]$, for $n \geq 4$, is the famous annulus theorem due to Kirby and Quinn. Why doesn't the annulus theorem follow easily from this exercise?