

## TOPOLOGICAL MANIFOLDS || PROBLEM SHEET 5

**Problem 1.** Let  $X$  and  $Y$  be compact metric spaces with  $X \times \mathbb{R}$  homeomorphic to  $Y \times \mathbb{R}$ . Then  $X \times S^1$  is homeomorphic to  $Y \times S^1$ .

Hint: let  $h: X \times \mathbb{R} \rightarrow Y \times \mathbb{R}$  be a homeomorphism, and consider the two product structures on  $Y \times \mathbb{R}$ , the intrinsic one and the one coming from  $h(X \times \mathbb{R})$ . Use a push-pull construction (repeated infinitely many times) to create a periodic homeomorphism  $H: X \times \mathbb{R} \rightarrow Y \times \mathbb{R}$ , i.e. for some  $p \in \mathbb{R}$ ,  $H(x, t) = H(x, t + p)$  for all  $t \in \mathbb{R}$ ,  $x \in X$ .