MATH 4161 Algebraic Topology IV || Epiphany 2019: Cohomology, products, and manifolds

Class Monday 1500 – 1600, CM221 and Thursday 0900–1000 CM107.
Problem class weeks 13,15,17,19, Wednesday 1200-1300 CM107.

Professor: Mark Powell
Office: CM106.
email: mark.a.powell@durham.ac.uk
Office hours: Tuesdays 1400-1500 and 1600-1700.

Plan for the course: The aim of the course is to learn about cohomology of spaces. After understanding its basic properties and the relationship between cohomology and homology, we will study the cup product. The cup product enables us to upgrade cohomology from a group to a ring. The ring structure captures deep information about a space. Then we will apply the cup product to the study of manifolds. There is another product called the cap product, that is used in the definition of Poincaré duality, which is a fundamental property special to manifolds. The material builds up towards a applications to the topology of manifolds.
Here is the rough plan for the course by week.

(1) Review of manifolds, homology groups of manifolds, orientation, fundamental classes, degree of maps between manifolds.
(2) Cochain complexes, cochain maps, cohomology of spaces – singular and CW.
(3) Properties of cohomology: relative groups, long exact sequences, pullback maps, excision, Mayer-Vietoris, cellular equals singular.
(4) Ext groups and the universal coefficient theorem.
(5) Cup product.
(6) Examples of the cup product.
(7) Cap product.
(8) Poincaré duality.
(9) Intersection and linking pairings of manifolds.
(10) Applications of intersection and linking pairings.

Lecture notes: I expect you to attend lectures and take your own notes during the lecture. If you are absent, I expect you to copy the notes of a friend or acquaintance.

There are extensive lecture notes of Stefan Friedl linked below. We are aiming to cover material from Chapters 33, 42, 44, 45, 51, 53, 54, 55, 56, 59, 62, 63. The notes give extremely detailed proofs so we certainly won't be able to cover all of them. They also go back to the beginning of topology, should you need a reminder of anything from Topology III or Michaelmas term Algebraic topology.

Homework: Homework will be due in fortnightly in lecture (I will announce the dates in due course) and we will discuss the solutions in the problems classes. I encourage you to discuss the problems in pairs or in small groups, but in that case each person must produce their own version of the solution.

Office hours: Please come and see me in my office hours to discuss the course and introduce yourself. We can negotiate the times of office hours to try to suit as many people's schedules as possible. Outside office hours times, it's usually a good idea to email me in advance to arrange an appointment.

You are also encouraged to attend the departmental drop-in sessions organised by the undergraduate director Steve Abel. I will be there so please give me something to do other than eat biscuits and gossip.

Learning outcomes: The exam will mostly consist of applications of the theory to examples. I will tell you which bookwork I expect you to be able to reproduce. Here are the intended learning outcomes. You will hopefully be able to:

- State and apply:
  (1) The long exact sequence of a pair in homology and cohomology.
  (2) The Mayer Vietoris sequence in homology and cohomology.
  (3) The Universal Coefficient Theorem.
  (4) The Künneth theorem.
  (5) The Poincaré duality theorem.
- Compute the homology and cohomology of some simple spaces using CW complexes and Mayer-Vietoris.
- Compute Ext groups and apply them to relate homology and cohomology.
- Apply functoriality of cup and cap products to investigate degrees of maps, and whether certain spaces are homotopy equivalent.
- Define and compute cup products for some simple spaces, especially surfaces and projective space.
- Compute the cup product of product spaces.
- Apply the Universal Coefficient Theorem and Poincaré duality to compute homology and cohomology groups of manifolds.

Please talk to me or email me to discuss any special circumstances that I ought to be aware of.