Holy smoke! Kiyoshi's argument seems to me to be correct. Here it is, fleshed out a bit.

Theorem (K. Igusa). *Suppose that M is a compact manifold (of any dimension) and $g : M \times S^1 \to M \times S^1$ is an isomorphism (diffeo, homeo, PL, ... Below I'll use generic terminology) such that*

1. *$g_{|}M \times \{1\}$ = Identity, and*
2. *g is isotopic to the Identity (of $M \times S^1$).*

Then g is isotopic to the Identity keeping $M \times \{1\}$ fixed (throughout the isotopy).

The point is, the isotopy of condition 2) may not keep $M \times \{1\}$ fixed. But in fact you can arrange that it does.

Remark. The only property of M that is used, aside from compactness, is that the Isotopy Extension Theorem holds for ('suitable' subsets of) it. So in fact M could be a compact polyhedron, or even ... . Also, it seems to me that in fact M need not be compact, by using classical 'majorant' arguments..

Proof of the Theorem. Regard S^1 as the interval [-1,1] with its endpoints identified (to a point called $1 \in S^1$). All motions/isotopies below are to be regarded as motions of $M \times S^1$ (or subsets thereof). They will be described either as isotopies of $M \times S^1$ which always are fixed on $M \times \{1\}$, or as isotopies of $M \times [-1, 1]$ which always are fixed on $M \times \{-1, 1\}$. We'll use whichever option is clearer and/or more natural at that point.

I'll break the proof into four steps (Compressing/Squeezing, Extending, Sliding, and Recovering/Undoing(the damaged region)). We begin with $g = g_a$, and produce in turn four additional isomorphisms $g_x : M \times S^1 \to M \times S^1$, each fixed on $M \times \{1\}$, for $x = b, c$, d and e, with g_e = Identity. Also we construct four (ambient) isotopies of $M \times S^1$ which 'join' the successive pairs of these g_x s. These isotopies are isomorphisms $G_{xy} : M \times S^1 \times I \to M \times S^1 \times I$ which preserve/respect the $I = [0, 1]$ coordinate, with each G_{xy} fixed on $M \times \{1\} \times I$, such that $G_{xy}|M \times S^1 \times 0 = g_x$ and $G_{xy}|M \times S^1 \times \{1\} = g_y$, for $xy = ab, bc, cd$ and de. The desired isotopy of the Theorem is then obtained simply by 'stacking' together these G_{xy}s.

Step 1) Enlarging the window where g = Identity.
Letting $g_a = g : M \times [-1, 1] \to M \times [-1, 1]$, isotope g_a to become g_b by enlarging (blowing up) $\{\pm 1\}$ to become $\pm[\epsilon, 1]$, so that g_b is the identity on $M \times \pm[\epsilon, 1]$ and $g_b|[-\epsilon, \epsilon]$ is a homothetically-squeezed (i.e. ϵ-scalar-conjugated) copy of g_a, with $\epsilon = 1/N$, N large, to be determined in the next step.

Step 2) Isotoping g_b to an isomorphism g_c which is the identity on $M \times [-\epsilon, \epsilon]$ (but will fail to be the Identity on $M \times \pm(\epsilon, 1)$).

To begin, lift the presumed isotopy G of hypothesis 2) to the cover $M \times R^1 \times [0,1]$ to obtain

$$\tilde{G} : M \times R^1 \times [0,1] \to M \times R^1 \times [0,1]$$

with $\tilde{G}|M \times R^1 \times \{1\} =$ Identity.

Choose N large enough so that $\tilde{G}(M \times [-1,1] \times [0,1]) \subset M \times (-N,N) \times [0,1]$, where here we regard $S^1 = R/2Z$, with fundamental domain $[-1,1] \subset R^1$. Then, letting $\epsilon = 1/N$ we 'squeeze' (= conjugate by multiplication by N in the R^1 factor) the restriction $\tilde{G}|M \times [-1,1] \times I \to M \times (-N,N) \times I$ to become an isotopy $H : M \times [-1/N, 1/N] \times I \to M \times (-1,1) \times I$. Next, extend H to isotopy $H_+ : M \times ([-1/N, 1/N] \cup \{1\}) \times I \to M \times S^1 \times I$ by defining $H_+|M \times \{1\} \times I =$ Identity. Now use the Isotopy Extension Theorem to extend H_+ to an isotopy $G_{bc} : M \times S^1 \times I \to M \times S^1 \times I$ with $G_{bc}|M \times \pm[1/N, 1] \times 0 =$ identity. Let $g_c := G_{bc}|M \times S^1 \times \{1\}$. Note that $g_c|M \times ([-1/N, 1/N] \cup \{1\}) =$ Identity, but we (essentially) have no idea what g_c looks like on the rest of $M \times S^1$. We do know (to be used in Step 4) that $G_{bc}|M \times 1/N \times I = \tau \circ G_{bc}|M \times -1/N \times I$, where $\tau : M \times S^1 \times I \to M \times S^1 \times I$is translation(rotation) by $2/N$ in the S^1 factor.

Step 3) Isotoping g_c to an isomorphism g_d which is the identity on $M \times [1-2/N, 1]$ (but not necessarily on $M \times (-1, 1-2/N)$).
All of the activity/motion during this Step takes place in the subset $M \times (-1/N, 1) \subset M \times S^1$. The idea is simply to move by isometric isotopy the subset $M \times (1/N, 1)$, on which g_c is unknown, gradually downward to become $M \times (-1/N, 1-2/N)$, always during this motion applying in the natural way g_c to this moving block. More precisely, for $(x,s,t) \in M \times [-1,1] \times I$ let

$$G_{cd}(x,s,t) = \begin{cases} (g_c(x,s),t) & \text{if } s \in [-1,-1/N], \text{ and} \\ (g_c(x,s+2t/N),t) & \text{if } s \in [(1-2t)/N, 1-2t/N], \text{ and} \\ (x,s,t) & \text{otherwise.} \end{cases}$$

Let $g_d = G_{cd}|M \times S^1 \times \{1\}$, and note that $g_d|M \times [1-2/N, 1] =$ Identity.

Step 4) Isotoping g_d to $g_e =$ Identity (completing the proof).
Now the idea is to use the 'covered' portion of the ambient isotopy of Step 2, that is the portion of G_{bc} defined on $M \times \pm(1/N, 1) \times I$, suitably adapted, to 'undo' the unknown part of g_d, that is to isotope $g_d|M \times [-1, 1-2/N]$ to the Identity rel $M \times -1, 1-2/N$, thereby isotoping $g_d : M \times S^1 \to M \times S^1$ to the Identity fixing $\{1\}$. In detail:

For $(x,s,t) \in M \times [-1,1] \times I$ first define

$$G'_{de}(x,s,t) = \begin{cases} G_{bc}(x,s,t) \text{ if} & s \in [-1,-1/N], \text{ and} \\ \tau^{-1} G_{bc} \tau(x,s,t) & \text{if } s \in [-1/N, 1-2/N] \text{ and} \\ (x,s,t) & \text{if } s \in [1-2/N, 1]. \end{cases}$$

where τ was defined in Step 2, as $\tau(x, s, t) = (x, s + 2/N, t)$.

Note that $G'_{de} lM \times S^1 \times 0 =$ Identity and $G'_{de} lM \times S^1 \times \{1\} = g_d$. Now define $G_{de}(x, s, t) = G'_{de}(x, s, 1 - t)$

End of proof.

Absolutely amazing!

When Danny first told us of this result (before it was posted) I thought of Step 1, and also realized that Step 2 was possible. But I couldn't see how that would help, nor could I see any other possibilities. And so I was unconvinced of the claim.

Kiyoshi's Steps 3-4 are brilliant. So simple, yet so consequential. Just like many other proofs of this ilk that we old-timers grew up with.

Bob