MAT993D: Exercise sheet 4

Exercise 1. Prove that $\pi_1^S \cong \mathbb{Z}_2$ using the Pontryagin-Thom construction. Describe a generator of this group.

Exercise 2. Compute the intersection forms of $S^2 \times S^2$, $S^1 \times S^1$ and $\mathbb{CP}^2$ and $L(p, q_1, q_2)$.

Exercise 3. Prove that the smash product $S^k \wedge S^l$ is homeomorphic to S^{k+l} and that the join $S^{k-1} * S^{l-1}$ is homeomorphic to S^{k+l-1}.

Exercise 4. Describe the regular homotopy classes of immersions $S^1 \to S^2$.

Exercise 5. Prove that the intersection form λ and the self-intersection number μ on $I_k(M)$ have the properties claimed in lectures.

Exercise 6. Prove that there is a homotopy equivalence $BO(k) \simeq V_n(\gamma_{k+n})$, where $\gamma_m \to BO(m)$ is the universal vector bundle, and for a vector bundle $E \to B$, $V_n(E)$ is the fibre bundle of orthonormal n-frames in E, known as the frame bundle.

Exercise 7. Prove that the two versions of quadratic form described in lectures are equivalent.

Exercise 8. Prove that for a f.g. projective module P, we have $P \cong P^{**}$.