**Topology of manifolds – surgery theory**
Winter 2016: Tuesday and Thursday 1500–1630.

**Professor: Mark Powell**
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Office hours: by appointment.

**Plan for the course:**

The aim is to introduce the theory of surgery, which tries to classify manifolds, especially those of dimension 5 and above. It enables us to determine whether one can find a manifold within a prescribed homotopy type, and if so, how many such manifolds can be found. We will begin with Smale's $h$-cobordism theorem, whose proof in 1959 sparked a burgeoning of high dimensional manifold topology. Then we will cover a number of important topics in algebraic topology, that will serve us all well to be acquainted with in our future research careers. These topics will form the basis of our study of surgery theory, the goal of which is to prove the surgery exact sequence. We will then try to cover some applications of the theory, and discuss its rôle in the topology of 4-manifolds.

(i) Introduction to manifolds and the surgery programme to classify them.

(ii) Morse theory, handle decompositions and the $h$-cobordism theorem. The Whitney trick.

(iii) Simple homotopy theory and the $s$-cobordism theorem.

(iv) Cohomology, basic homological algebra such as universal coefficients, the Eilenberg-Zilber theorem and Poincaré duality.

(v) Fibre and vector bundles. A spectral sequence.

(vi) Cobordism theory and some homotopy theory.

(vii) The Spivak normal fibration and the normal invariants.

(viii) The surgery kernel and surgery below the middle dimension.

(ix) Intersections and self-intersections, definitions of $L$-groups and computation in some cases.

(x) The surgery exact sequence, Wall realisation.

Additional topics, possibilities for student talks: applications to exotic spheres, classification of simply connected 4-manifolds. The Borel, Novikov and Farrell-Jones conjectures. The total surgery obstruction. Kreck's modified surgery. Homology surgery theory, $\Gamma$-groups. Topics in 4-manifolds such as Kirby calculus, Rochlin's theorem on spin 4-manifolds. Computation of $\Omega_3, \Omega_4$. Freedman's theorem on slice knots.

**Prerequisites:**

I will assume that you know, or can quickly recall, the basics of algebraic topology as in the first 3 chapters of Hatcher: fundamental group and covering spaces, singular and cellular homology, homotopy invariance, relative homology groups, long exact sequence of a pair, Mayer-Vietoris, excision, homotopy groups, 5-lemma, snake lemma, Hurewicz and Whitehead theorems.

**Evaluation:**

Attendance and engaging with the course, 65% (this is a C). Students will give a 30–45 minute talk on a topic related to the course 20% (that means you get an A if you also give a talk). Exercises will be given during the course, careful solutions to some of these will form a take home final, 15%.

**References:**

Course material at: http://www.math.uqam.ca/∼powell/TopologyOfManifolds.html

I will mostly be using the following books. Links to electronic copies can be found on my website, where further references will also be added as required.


1. A.A. Ranicki: Algebraic and geometric surgery.
2. D. Crowley, W. Lück, T. Macko: Surgery theory foundations.
3. J. Davis, P. Kirk: Lecture notes on algebraic topology.
4. A. Hatcher: Algebraic topology.
5. C.T.C. Wall: Surgery on compact manifolds.
6. J.W. Milnor: Morse theory and Lectures on the $h$-cobordism theorem.
7. G.E. Bredon: Topology and Geometry.