Seminar || Topological manifolds II

University of Bonn,

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TABLE 1. Schedule

Date	Topic	Speaker
	Foundations	
April 21	Manifolds embed in Euclidean space and are ENRs	Raphael Floris
	Wildness	
April 28	The Alexander horned sphere and Bing's hooked rug	Franca Lippert
May 5	The double suspension of the Mazur homology sphere is S^5	Fadi Mezher
	h-cobordism	
May 12	Adding a boundary to a noncompact manifold	Alice Merz
May 19	*No seminar: Dies academicus*	
May 26	*No seminar: Pfingstferien*	
	Microbundles	
June 2	Homotopy invariance of pullback	Cara Hobohm
	PL-ing obstructions	
June 9	A non-PL-able manifold	Ekin Ergen
June 16	Rochlin's theorem	Christian Kremer
	Torus trick	
June 23	Explicit immersion of punctured n-torus in $\mathbb{R}^n$	Ben Ruppik
June 30 July 7	Torus trick for surfaces and 3-manifolds	Daniel Galvin & Weizhe Niu
	Application of Edwards-Kirby	
July 14	Counting topological manifolds	Magdalina von Wunsch Rolshoven
	Engulfing	
July 21	Unique PL and smooth structures on $\mathbb{R}^n$	Diego Santoro

Overview. A manifold is a Hausdorff, paracompact topological space that is locally homeomorphic to $\mathbb{R}^n$. One frequently encounters manifolds with additional structure, such as smooth, Riemannian, or symplectic. The aim of this seminar is to learn about unadulterated topological manifolds, and whether they admit piecewise linear structures.

The topics of the seminar are closely related to the lecture course in Winter Semester 2020/21. However, participation in that course is not a prerequisite.

We have grouped the topics in themes, and suggest working on talks in teams. It will not be necessary to have understood all the previous talks to prepare a talk, or to follow a talk, although there will hopefully be connections between the talks within a theme.

Bonn students will have priority in topic selection, but we strongly welcome talks from others, especially participants in the Winter Semester course.

Logistics. The following is primarily relevant for Bonn students.

- The "exam" for the seminar is your talk. As you prepare, please get in touch with us as questions arise. We hope to talk with each of you at least once before your talk (or more, as needed or desired).
- Talks are 90 minutes, with a 15 minute break in the middle, to be delivered over Zoom.
- To "qualify" for the exam, please prepare a typed document with the contents of your talk. In general the document will contain more details than what you can present in the talk.
- The typed document must be in LaTeX, using a template we will provide. (With your consent) we would like to include these writeups in the "lecture notes" project started in the winter semester course – results and definitions in those notes can be freely assumed and referenced. The final version from the course is here: https://maths.dur.ac.uk/users/mark.a.powell/TM-lecture-notes-current.pdf
- Within two weeks (in either direction) of your talk, please send us a draft for your writeup. We will return the draft to you with comments, and the final version will be due two weeks after that.
- Within "theme" groups, e.g. Foundations or Wildness, feel free to collaborate in preparing your talks and/or writeups. Interaction beyond thematic groups is also strongly encouraged!

Resources for assigned talks.

(1) Manifolds are ENRs – [Dol95, Chapter IV.8][Han51, Theorem 3.3]
(2) Counting topological manifolds [CK70]
(3) Alexander horned sphere and Bing's hooked rug [DV09]
(4) Double suspension of Mazur homology sphere – [Fer, Edw]
(5) Boundaries for noncompact manifolds [BLL65]
(6) Homotopy invariance of pullback of microbundles – [Mil64]
(7) A non-PL-able manifold – [KS77, Annex C]
(8) Rochlin's theorem [FK78, Kir89]
(9) Explicit immersion of punctured n-torus in $\mathbb{R}^n$ – check course website
(10) Torus trick for surfaces and 3-manifolds [Ham76, Hat]
(11) $\mathbb{R}^n$ has a unique PL and smooth structure except in dim 4 – [Fer, Sta62].

Remaining topics.

Collaring.

(12) Brown's proof of collaring, collaring for noncompact boundaries [Bro62], Casson.
(13) Uniqueness of collars of boundaries [Arm70].

Wildness.

(14) The Whitehead manifold and manifold factors [Dav07].

h-cobordism.

(15) Proof of (relative) h-cobordism theorem, sketch proper version – [Sma62, Sie70], see also [Sco05].

Microbundles.

(16) Stable existence and uniqueness of normal microbundles – Hirsch (team of two if desired).

Product structure theorem applications.

(17) Topological transversality [KS77].

(18) Simple homotopy type for topological manifolds [KS77].

PL-ing obstruction.

(19) Casson's construction of a fake $D^3 \times T^n$ [KS77, Essay IV, Appendix B].

(20) Normal bundles in codimension two – Kirby Siebenmann (hard).

Applications of Edwards-Kirby

(21) Immersion theory – Haefliger-Poénaru (warning, en français).

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