1

(a) Let X be a complex manifold. Explain how to associate to X an *almost complex structure*

$$J_X : TX \to TX,$$

and show that J_X is well-defined.

(b) Let $\varphi : X \to Y$ be a smooth map between complex manifolds X and Y, and suppose J_X and J_Y are the almost complex structures associated with X and Y respectively. Show that φ is holomorphic if and only if

$$d\varphi \circ J_X = J_Y \circ d\varphi.$$

(c) Give examples of smooth maps

$$\varphi_1 : \mathbb{P}^1 \to \mathbb{P}^1,$$
$$\varphi_2 : \mathbb{P}^1 \to \mathbb{P}^1,$$

such that φ_1 is holomorphic and φ_2 is not holomorphic.