K-STABILITY AND KÄHLER GEOMETRY

CAMBRIDGE, SEPTEMBER 7TH - 9TH, 2021

All the talks will be held in Bateman Auditorium, Gonville & Caïus College.

Tuesday, September 7th.

- 12:00- 13:00 Lunch and registration (Bateman Room)
- 13:00 - 14:00, Hamid Abban (Loughborough University)
- 14:00 Coffee (Bateman Room)
- 14:30-15:30, Stuart Hall (Newcastle University)
- 15:30 Break (Bateman Room)
- 16:00-17:00, Rémi Reboulet (Université Grenoble-Alpes)
- 17:00 Wine reception (Bateman Room)

Wednesday, September 8th.

- 9:30-10:30, Jack Rogers (University of Manchester)
- 10:30 Coffee (Bateman Room)
- 11:00-12:00, Simon Jubert (Université du Québec à Montréal/Université de Toulouse)
- 12:00 Lunch (Hall)
- 13:30-14:30, Jesus Martinez-Garcia (University of Essex)
- 14:30 Coffee (Bateman Room)
- 15:00-16:00, Anne-Sophie Kaloghiros (Brunel University London)
- 16:00 Break (Bateman Room)
- 16:30-17:30 Lars Martin Sektnan (University of Gothenburg)
- 19:10 Conference dinner (Senior Parlour)

Thursday, September 8th.

- 9:00-10:00, John McCarthy (Imperial College London)
- 10:00 Coffee (Bateman Room)
- 10:30-11:30, Carlo Scarpa (SISSA)
- 11:30 Break (Bateman Room)
- 12:00-13:00 Frances Kirwan (University of Oxford)
- 13:00 Lunch (Hall)

Program

K-stability via double filtration
Hamid Abban (Loughborough University)
Abstract: I will discuss some of the known methods for checking K-(semi)stability of Fano varieties. I will pay particular attention to a new method that relies on double filtration, illustrated with several examples. This is joint work with Ziquan Zhuang.

An application of Kähler Quantisation to molecular similarity searching
Stuart Hall (Newcastle University)
I will report on joint work with Rachael Pirie, Daniel Cole, Matthew Forshaw, and Tommy Murphy using ideas from Kähler geometry to discretise surfaces immersed in $\mathbb{R}^3$ in such a way as to allow for an efficient and effective comparison (i.e. how similar are two of them?). We will focus on surfaces associated to molecules and the applications this has to virtual drug discovery. I'll try to give a brief overview of the theory as well as describe the various considerations involved in actually carrying it out for the associated surfaces. If I have time, I'll also talk about how the framework allows for fast approximation of the low lying spectrum of the Laplacian; this leads to other discretisations rooted in spectral geometry.

Non-Archimedean plurisubharmonic geodesics and complex limits.
Rémi Reboulet (Université Grenoble-Alpes)
Abstract: We study the space of finite-energy plurisubharmonic metrics on the Berkovich analytification of an ample line bundle on a variety over a non-Archimedean field. We discuss the construction of plurisubharmonic geodesics in this space, in parallel with classical results in the complex setting. We then see how specific examples of such geodesics can be interpreted as limits of complex plurisubharmonic geodesics on degenerations of complex manifolds.

K-stability of smooth Fano SL2-threefolds
Jack Rogers (University of Manchester)
Abstract: There has been much interest in K-stability since it was shown to be equivalent to the existence of Kähler-Einstein metrics by Chen-Donaldson-Sun. The theory of K-stability is now well developed, but practical methods to check whether a given variety is K-stable are hard to come by. Equivariant K-stability, introduced by Datar-Székelyhidi, makes finding such criteria easier for varieties with large automorphism groups. If an algebraic group G acts on a variety X, the complexity of the action is the minimal codimension

in X of the orbits of a Borel subgroup B of G (e.g. if T is a torus, the complexity zero T-varieties are the toric varieties). Conditions for K-stability have been found for toric varieties by Wang-Zhu, for complexity one T-varieties by Ilten-Süss and for all complexity zero varieties by Delcroix. We will discuss the combinatorial description due to Timashev of complexity one G-varieties, and describe a practical method to check K-stability in the particular case of smooth Fano SL2-threefolds. In particular, this method proves the K-stability of several varieties not previously known to be K-stable, e.g. projective 3-space blown up along three disjoint lines. This is joint work with Hendrik Süss.

A Yau-Tian-Donaldson type correspondence on a class of toric fibrations

Simon Jubert (Université du Québec à Montréal/Université de Toulouse)

Abstract : I will present a Yau-Tian-Donaldson type correspondence, expressed in terms of Delzant polytope, concerning the existence of extremal Kähler metrics on a class of toric fibrations called "semi-simple principal toric fibration". After introducing/recalling the notion of weighted cscK metric in the sense of Lahdili, I will sketch the construction of semi-simple principal toric fibration. We will see that the extremal Kähler metrics on the total space correspond to weighted constant scalar curvature Kähler (cscK for short) metrics on the corresponding toric Kähler fiber. We will then characterize the existence of such a weighted cscK metric in terms of weighted uniform K-stability of the corresponding Delzant polytope. If time allows, we will apply the previous result to study the existence of extremal metrics on some projective plane bundles over an elliptic curve.

The Calabi problem for Fano 3-folds

Jesus Martinez-Garcia (University of Essex), Talk I
Anne-Sophie Kaloghiros (Brunel University London) Talk II

Abstract: We will discuss progress on the Calabi problem for Fano 3-folds. The 105 deformation families of smooth Fano 3-folds, were classified by Iskovskikh, Mori and Mukai. We determine whether or not the general member of each of these 105 families admits a Kähler-Einstein metric. In some cases, it is known that while the general member of the family admits a Kähler-Einstein metric, some other member does not. This leads to the problem of determining which members of a deformation family admit a Kähler-Einstein metric when the general member does. This is accomplished for most of the families, and we will discuss some examples and present a conjectural picture for some of the remaining families. This is a joint project with Carolina Araujo, Ana-Maria Castravet, Ivan Cheltsov, Kento Fujita, Constantin Shramov, Hendrik Süss and Nivedita Viswanathan.

Constructing extremal Kähler metrics on the total space of destabilising test configurations

Lars Martin Sektnan (University of Gothenburg)

Abstract: In this talk, I will discuss a new construction of extremal Kähler metrics. These metrics are produced on the total space of certain destabilising test configurations, in adiabatic Kähler classes. The construction produces many new examples of extremal metrics. This is joint work with Cristiano Spotti.

Bridgeland stability and Z-critical connections

John McCarthy (Imperial College London)

Abstract: Bridgeland stability is a general notion of stability for complexes of sheaves, and it is predicted that this notion of stability should correspond to a differential-geometric notion of an extremal metric or connection, such as the recent "deformed Hermitian Yang-Mills equations." In this talk I will introduce a notion of extremal connection on a holomorphic vector bundle, which we call a Z-critical connection, which should correspond to a Bridgeland stability condition. This is made precise in a certain limit (the "large volume limit") in which Bridgeland stability simplifies to a type of stability of vector bundles we call Z-stability. In this limit we prove a concrete correspondence between Z-stable vector bundles and solutions of the Z-critical equations, which will be described in the talk. Away from this limiting situation we show a correspondence for line bundles over complex surfaces and speculate about the general situation. This is joint work with Lars Sektnan and Ruadhai Dervan.

K-stability and the Hitchin-cscK system

Carlo Scarpa (SISSA)

Abstract: The Hitchin-cscK system is an analogue of Hitchin's Higgs bundle equations on the category of polarized varieties. The system is obtained by adding a suitable Higgs field, i.e. a first-order deformation of the complex structure, to the cscK equation. In the first part of this talk, we will discuss some general properties of the Hitchin-cscK system, focusing on a generalization of K-stability that conjecturally describes the existence of solutions to the system. In the second part, we will describe more concrete results in the toric case, where the general theory greatly simplifies.

Moment maps for nonreductive group actions in Kähler geometry.

Frances Kirwan (Oxford University)

Abstract: When a complex reductive group G acts linearly on a projective variety X, the GIT quotient $X//G$ can be identified with a symplectic quotient of X by a Hamiltonian action of a maximal compact subgroup K of G. Here the moment map takes values in

the (real) dual of the Lie algebra of K, which embeds naturally in the complex dual of the Lie algebra of G (as those complex linear maps taking real values on Lie K). The aim of this talk is to discuss an analogue of this description for GIT quotients by suitable non-reductive actions, where the analogue of the moment map takes values in the complex dual of the non-reductive group. This is joint work with Gergely Berczi.