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Rotation and magnetism of solar-like stars: from scaling laws to spot-dynamos

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Abstract. The Sun is the archetype of magnetic star and its proximity coupled with very high accuracy observations has helped us understanding how solar-like stars (e.g with a convective envelope) redistribute angular momentum and generate a cyclic magnetic field. However most solar models have been so fine tuned that when they are applied to other solar-like stars the agreement with observations is not good enough. I will thus discuss, based on theoretical considerations and multi-D MHD stellar models, what can be considered as robust properties of solar-like star dynamics and magnetism and what is still speculative. I will derive scaling laws for differential rotation and magnetic energy as a function of stellar parameters, discuss recent results of stellar dynamo models and define the new concept of spot-dynamo, e.g. global dynamo that develops self-consistent magnetic buoyant structures that emerge at the surface.

Keywords. Sun: convection, rotation, magnetism; dynamo; stellar magnetism and rotation

1. Introduction

The Sun exhibits fascinating magnetic phenomena, with sunspot emergence, flares, prominences and CME's most of which varies in number and intensity during the 11-yr activity cycle. Being able to understand the origin of such a large variety of magnetic manifestations and their link to the underlying solar dynamo has been challenging. Many observations show for instance that the symmetric (quadrupolar-like) and antisymmetric (dipolar-like) dynamo families come into play to modulate the 11-yr cycle, make one hemisphere lag the other during reversals (DeRosa *et al.* (2012)) and sometimes even lead to grand minima of activity (Tobias(1997)). It is thus important in order to progress in our current understanding of the solar dynamo to characterize how dynamo actions varies as a function of stellar parameters. Thanks to improved instrumentations, observations of the magnetism of solar-type stars, i.e. stars possessing a deep convective envelope and a radiative interior (late F, G, K and early M spectral type) are becoming more and more available Giampapa (2005). One difficulty of such observational programs is that they require long term observations since stellar cycle periods are likely to be commensurate to the solar 11-yr sunspots cycle period. Thanks to the data collected at Mount Wilson Observatory since the late 60's, such data is available (Wilson (1978), Baliunas *et al.* (1995)). Among the sample of 111 stars (including the Sun as a star) originally observed between the F2 to M2 spectral types, it is found that about 50% of the stars possess a cyclic activity, with cycle (starspot) periods varying roughly between 5 to 25 yrs, i.e. between half to twice the sunspot cycle period. They further indicate that among the inactive stars of the sample some are likely to be in a quiet phase (as was the Sun during the Maunder minimum). Overall activity cycles seem to be more frequent for less massive K stars than for F stars. More recent observational programs have been pursued that now even provide information on the field topology as a function of the rotation rate, such as the one using the Espadons and Narval instruments and the Zeeman Doppler

Imaging technique (Donati *et al.* (1997)). Applying this observational technique over a sample of four solar analogues with rotation rate Ω_0 varying from one to three times solar $\Omega_\odot$, Petit *et al.* (2008) have shown that the field amplitude increases as a function of the star's rotation rate and, more importantly, becomes more and more dominated by its toroidal component (modulo possible bias in the observational technique used). If such a trend is confirmed, i.e. that the field topology is becoming more toroidal with increasing rotation rate, it is a very important and instructive result and puts strong constraints on the dynamo models. In a more recent study Morgenthaler *et al.* (2011) have continued to monitor these stars over several years and have observed that some of them underwent a reversal of their global magnetic field, confirming the tendancy of solar-like stars to have time varying (cyclic?) global field polarity.

The systematic analysis of stellar magnetism data revealed that for solar type stars there is a good correlation between the cycle and rotation periods of the stars and that correlation is even stronger when using the Rossby number ($Ro = P_{rot}/\tau$) that takes into account the convection turnover time τ at the base of the stellar convective envelope (Noyes *et al.* (1984), Baliunas *et al.* (1996)). As the star rotates faster, its cycle period is found to be shorter. Typically, Noyes *et al.* (1984) found that $P_{cyc} \propto P_{rot}^n$, with $n = 1.25 \pm 0.5$. Based on an extended stellar sample Saar & Brandenburg (1999) and Saar (2002) have argued that there is actually two branches when plotting the cycle period vs the rotation period of the stars (results later confirmed by Böhm-Vitense(2007)). They make the distinction between the primary (starspot) cycle and Gleissberg or grand minima type modulation of the stellar activity. For the active branch they found an exponent $n \sim 0.8$ and for the inactive stars $n \sim 1.15$. It is also found that this correlation breaks at high rotation rate with the possible appearance of a super active branch. Recent progress based on asteroseismic data from Corot and *Kepler* have also started to put new constraints on stellar magnetism by increasing the number of observed stars (Mathur *et al.* (2013)) thanks to the change of frequency of oscillations (mostly acoustic modes) induced by magnetic field. There too evidence for cyclic activity are found (García *et al.* (2010)). It was also noticed that at very high rotation rate, the chromospheric (soft-X ray) activity level usually used as a good proxy for stellar magnetism, is saturating. The saturation of the X-ray luminosity seems to limit the validity of the scaling found at more moderate rotation rates (Pizzolato *et al.* (2003), Wright *et al.* (2011)). For G type stars this saturation is found for rotation rate above 35 kms^{-1}, for K type stars at about 10 kms^{-1} and for M dwarfs around 3-4 kms^{-1}, so about a Rossby number of 0.1. Note that the observed quantity Bf, where B is the field amplitude and f the surface filling factor, does not allow to distinguish if B actually saturates with Ω_0 or if only the filling factor f does (Reiners (2012)). It is thus also important to understand through dynamo simulations how stellar magnetic flux scales with rotation rate since it is telling us how the magnetic field generated by dynamo action inside the stars emerges and imprints the stellar surface and how it varies.

2. Stellar Dynamo: theoretical concepts

Stars possess a priori all the ingredients necessary to the development of a dynamo instability (Weiss (1994)), such as a large-scale shear (or differential rotation), turbulent motions, helicity (thanks to rotation and its associated Coriolis force) and low diffusivity. All these properties are favourable to the emergence by dynamo action of a magnetic field. Observations in the Sun and in most solar-like stars (which much less details of course), of phenomena such as starspots or flares, clearly hint to the presence of magnetic fields. Their temporal dependence and properties has led naturally to consider that the origin of this magnetism is indeed dynamo action. However, we also know that stellar magnetic activity manifests itself in a multitude of facets (irregular, cyclic, modulated), certainly indicating the presence of several types of dynamo or magnetism. Until the recent advent of massively parallel super computers, astrophysicists were especially interested with the cyclic and large-scale dynamo and developed simplified models based on mean field dynamo theory and have put forward the fundamental concept of α and ω effects (Moffatt(1978), Charbonneau (2010)).

Theoretical considerations to interpret stellar magnetism based on classical mean field α-ω dynamo models (Durney & Latour (1978), Baliunas *et al.* (1996), Montesinos *et al.*(2001)) naturally yield correlation between rotation rate and stellar activity. In particular it is found that both magnetic field generation and the dynamo number D (i.e. a Reynolds number characterizing the mean field α and ω dynamo effects used in the models) vary with the rotation period of the star $D \propto 1/Ro^2$. This is due to the fact that in these models both effects are sensitive to the rotation rate of the star. The ω-effect is a direct measure of the differential rotation $\Delta\Omega$ established in the star. It is well known both theoretically and observationally that the differential rotation in the convective envelope of solar-type stars is directly connected to the star's rotation rate Ω_0 (Donahue *et al.*(1996), Barnes *et al.*(2005), Ballot *et al.* (2007), Brown *et al.* (2008), Küker *et al.*(2011), Matt *et al.* (2011), Augustson *et al.* (2012), Gastine *et al.* (2013)). However, the exact scaling exponent n_r (i.e. $\Delta\Omega \propto \Omega_0^{n_r}$) is still a matter of debate among both the observers and the theoreticians, being sensitive to both the observational techniques used and to the modelling approach. Likewise since the α-effect is a parameterization of the mean electromotive force (*emf*), it was actually shown to be directly related to helical turbulence (Moffatt(1978), Pouquet *et al.*(1976)), thus naturally connected to the rotation rate of the star and the amount of kinetic helicity present in its convective envelope. So this explains why in α-ω dynamo model it is straigthforward to related rotation and dynamo action.

However the currently preferred solar dynamo model, e.g. the so called flux transport Babcock-Leigthon dynamo model (Dikpati *et al.* (2004)), relies not on the α-effect to regenerate the poloidal component of the magnetic field but on the so-called Babcock-Leighton effect (Babcock(1961), Leighton(1969)), e.g. the tendancy for active regions or sunspot bipoles to be tilted with respect to the east-west direction (Joy's law). So as the active regions decay away over several weeks, the poloidal component of the diffuse field plays the actual role of a source term. This tilt is thought to be due to the action of the Coriolis force during the rise and emergence of the toroidal structures as active regions (D'Silva & Choudhuri(1993)). So here too a simple link to rotation can be obtained. Note however that recent 3-D simulations in spherical shells with developed convection motions (Jouve *et al.*(2013), and references therein) indicate that this is not the only effect responsible for the observed tilt and that the twist and arching of the toroidal structures as well as the continuous action of the surface convection during the emergence have some influence on the resulting tilt. So it may not be as simple as anticipated to relate a Babcock-Leigthon like source term to rotation and one may anticipate a different scaling between D and Ro than is standard α-ω dynamos. Further another important ingredient in flux transport models is the large scale meridional circulation (MC) used to connect the surface source term generating the poloidal field to the region of strong shear at the base of the convection zone (i.e. the tachocline) where it will be subsequently sheared by the ω-effect in order to close the global dynamo loop (i.e. $B_{pol} \rightarrow B_{tor} \rightarrow B_{pol}$). The meridional flow (or "conveyor belt") thus plays an important role in setting the cycle period of the global dynamo in this class of models. As a direct consequence it is natural to ask how the meridional circulation amplitude and profile change with the rotation rate and how these may influence the magnetic cycle period. Several authors have thus looked at the influence of the meridional circulation on the butterfly diagram and activity cycle period (Dikpati *et al.*(2001), Charbonneau & Saar(2001), Nandy(2004), Jouve & Brun (2007), Nandy & Martens(2007)). They all reached the same conclusion: only a positive scaling of the amplitude of meridional flows with the rotation rate can reconcile the models with observations of magnetism of solar-like stars. Unfortunately as we will discuss in the next sections, 3-D simulations actually find the opposite, the meridional circulation actually weakens with faster rotation rates, and this has important consequences for current stellar dynamo model as demonstrated by Jouve *et al.* (2010). So while it is easy to find a link between magnetic field amplitude, cycle period and rotation rate, observations of stellar magnetism actually impose that these relationship follow very specific trends not necessarily fitting our current solar dynamo paradigm.

3. 2-D mean field models of stellar magnetism

As we have seen, the observational correlation between rotation and activity obtained by Noyes *et al.* (1984), Baliunas *et al.* (1996), Saar & Brandenburg (1999), Böhm-Vitense(2007) could be due to the influence of rotation on dynamo action in stellar convective envelopes. However, flux transport dynamo model are in difficulty because their cycle period P_{cyc} depends strongly on the meridional circulation amplitude (as well as its profile, Jouve & Brun (2007)):

$$P_{cyc} \propto \Omega_0^{0.05} s_0^{0.07} v_0^{-0.83} \tag{3.1}$$

As will be seen in §4 the meridional circulation is found to decrease with the rotation rate as $v_0 \propto \Omega_0^{-0.45}$. This is not intuitive as one could expect that the meridional circulation increases with the rotation rate. A careful study of the vorticity equation shows that it actually weakens with rotation rate as more and more kinetic energy is being transferred to longitudinal motions at the expense of meridional kinetic energy. The fact that in recent 3-D simulations the meridional circulation is found to weaken as the models is rotated faster directly implies that standard advection dominated flux transport dynamo models yield the opposite dependency with rotation than the one observed, e.g. activity cycles are found to be longer for faster rotating stars (Jouve *et al.* (2010)). This fact alone impose to revise our current dynamo paradigm for solar-like stars. One way is to shortcircuit the advection path by for instance adding more cells in latitude or increase the radial diffusion as was done in Jouve *et al.* (2010), Hazra *et al.*(2013). Considering several cells either in latitude and/or radius for the meridional circulation is actually in better agreement with numerical simulations of rotating convection zone at low Rossby number and seems to also be observed in the Sun (Zhao *et al.*(2013)). An alternative is to consider another transport process such as magnetic turbulent pumping. We will now discuss this new class of dynamo models in more details (see also Guerrero & de Gouveia Dal Pino (2008), DoCao & Brun (2011)).

Magnetic pumping refers to transport of magnetic fields in convective layers that does not result from bulk motion. One particular case is turbulent pumping. In inhomogeneous convection due to density stratification, convection cells take the form of broad hot upflows surrounded by a network of downflow lanes Miesch *et al.* (2008). In such radially asymmetric convection, numerical simulations show that the magnetic field is preferentially dragged downward (Tobias *et al.*(2001)). This effect has been demonstrated to operate in the bulk of the solar convection zone. A significant equatorward latitudinal component also arise when rotation becomes important, i.e. when the Rossby number is less than unity. Turbulent pumping speeds of a few ms^{-1} can be reached according to the numerical simulations of Käpylä *et al.* (2006). Therefore, its effects are expected to be comparable to those of meridional circulation.

In the mean field dynamo framework in order to model the global dynamo operating in solar-like stars, we start from the induction equation, which after the usual scales separation between mean and fluctuating fields, e.g. $\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$, becomes (Moffatt(1978)):

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle) + \nabla \times \langle \mathbf{v} \times \mathbf{b} \rangle - \nabla \times (\eta_m \nabla \times \langle \mathbf{B} \rangle). \tag{3.2}$$

A closure relation must then be used to express the mean electromotive force (*emf*) $\epsilon = \langle \mathbf{v} \times \mathbf{b} \rangle$ in terms of mean magnetic field, leading to a simplified mean-field equation. If the mean magnetic field varies slowly in time and space, the *emf* can be represented in terms of $\langle \mathbf{B} \rangle$ and its gradients

$$\epsilon_i = a_{ij} B_j + b_{ijk} \frac{\partial B_j}{\partial x_k} + ... \tag{3.3}$$

where a_{ij} and b_{ijk} are in the general case tensors containing the transport coefficients (for simplicity we dropped $\langle\rangle$). The tensors a_{ij} and b_{ijk} cannot, in general, be expressed from first principles due to the lack of a comprehensive theory of convective turbulence. In the kinematic regime where the magnetic energy is negligible in comparison to the kinetic energy, the most simple approximation is to neglect all correlations higher than second order in the fluctuations. This is the so-called first order smoothing approximation (FOSA); Charbonneau (2010). In most studies isotropic turbulence is assumed and the pseudo tensor a_{ij} reduces into a single scalar giving rise to the α-effect. However in the full tensor non-isotropic case for a, the *emf* can be expressed as:

$$\epsilon = (\alpha \mathbf{B} + \gamma \times \mathbf{B}) - \beta \nabla \times \mathbf{B} \tag{3.4}$$

where α is a scalar referring to the standard α-effect. The term γ is the turbulent pumping and β is defined such that $b_{ijk} = \beta \epsilon_{ijk}$ (with ϵ_{ijk} the Levi-Civita tensor) and represents the turbulent enhancement of magnetic diffusion. Note that as we work in the framework of Babcock-Leighton flux transport models, we will replace the $\alpha \mathbf{B}$ term by a non local source term S representative of flux emergence.

Stellar mean field dynamo models have been studied in the case of a shallow MC by Guerrero & de Gouveia Dal Pino (2008). We have expanded in DoCao & Brun (2011) their results by considering both a deeper MC and various rotation rates, applying this pumping dominated dynamo models to other stars. Under certain conditions we showed that turbulent pumping can shorten the advection path driven by MC. We refer to DoCao & Brun (2011) for detailed analytical expression of the turbulent pumping. On Figure 1 we show buttefly diagrams obtained with this new dynamo model. We clearly see that for faster rotation the cycle period is shorter and the butterfly diagram remains solar-like with the correct phase relationship between the poloidal and toroidal field components.

By studying a large range of paremeters, DoCao & Brun (2011) were able to derive the following dependancy for the cycle period P_{cyc}:

$$P_{cyc} \propto v_0^{-0.40} \gamma_{r0}^{-0.30} \gamma_{\theta 0}^{-0.15} \tag{3.5}$$

We found that the turbulent pumping becomes a major player in setting the magnetic period, but its influence is not as large as in Guerrero & de Gouveia Dal Pino (2008). First, the MC is still the dominant effect and the radial pumping component is not as important. Second, the effect of γ_θ is not negligible. This supports the idea that the latitudinal advection process, and especially at the BCZ, is an important ingredient in advection dominated BL models, capable of transporting the toroidal magnetic field from the pole toward the equator. This difference may come from their choice of a shallow MC with almost zero velocity at the BCZ.

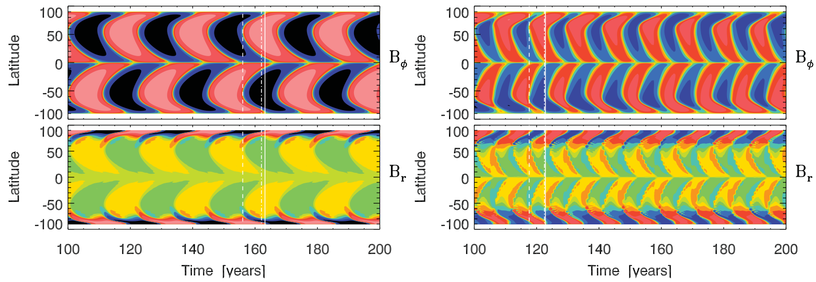

Figure 1. Butterfly diagram for 2 representative cases : $0.7\Omega_\odot$ and $3.0\Omega_\odot$. Both figures share the same color scale : between -510^3G and 510^3G for B_r and between -910^5G and 910^5G for B_ϕ. We also show the phase relations between B_{pol} and B_{tor}, (DoCao & Brun (2011))

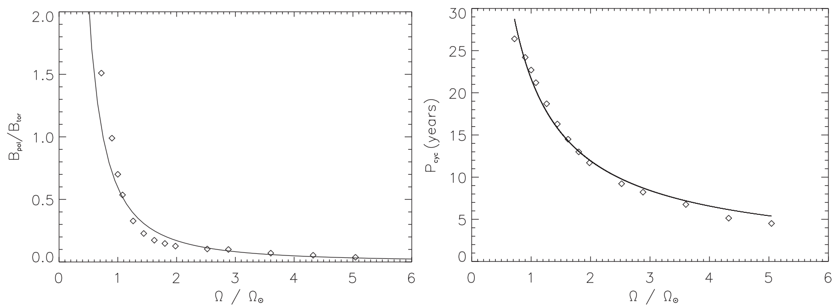

Figure 2. Left: $B_{\rm pol}/B_{\rm tor}$ ratio as function of the rotation rate. Solid line is a least square fit of the data. Right: Magnetic cycle period as function of the rotation rate in models including turbulent pumping. Solid line is a least square fit of the simulated data (DoCao & Brun (2011))

On Figure 2 we show the ratio of the poloidal to toroidal field and the cycle period as a function of the star's rotation rate. We clearly find the observed tendancy of a more and more dominant toroidal component and of shorter cycle. A simple look at the scaling law 3.5 gives that if we want to recover these observational trends, in particular the shorter cycle period (again, we assume that $v_0 \propto \Omega^{-0.45}$), and assuming that γ_r/γ_θ remains constant, the pumping effect should roughly scales as Ω_0^2. Such a scaling may be too extreme and only systematic 3-D numerical simulations will tell us if this is the case or not. Nevertheless, pumping dominated stellar dynamos are a plausible solution to explain observations.

Note that recent observations of the Sun and of solar analogues also point to the important role played by the symmetric family dynamo modes (such as the axisymmetric quadrupole) and that succesful dynamo models must also possess symmetric and not just antisymmetric equatorial symmetry. Indeed DeRosa *et al.* (2012) have shown that in the Sun the symmetric modes contribute to about 25% of the overall magnetic energy and that during reversals the quadrupolar mode actually dominates. This is certainly at the origin of the time lag of 1 to 2 years between the north and southern hemisphere in the Sun, since a dipole plus a quadrupole of equal amplitude would lead to an hemispherical dynamo. During grand minima activity phase, such as during the Maunder minimum, an hemispherical state with mostly sunspots in the southern hemisphere has been observed, there too, pointing for a strong contribution of the symmetric dynamo family Tobias(1997). It is certainly also the case that solar-like stars do not possess a purely antisymmetric state of their magnetic field and that both dipolar and quadrupolar-like symmetries are found Petit *et al.* (2008). Simple kinematic dynamos usually do not couple both dynamo familiies due to simple symmetry considerations of their main ingredients.

Only the introduction of asymmetric flows or source terms at the level of 0.1% can couple the families to the adequate level (DeRosa *et al.* (2012)). An alternative is to use dynamo coefficients deduced from 3-D numerical simulations as in Dubé & Charbonneau(2013). Indeed nonlinear coupling between both symmetric and antisymmetric dynamo families can easily be achieved in 3-D simulation of convective dynamos Strugarek *et al.* (2013).

4. 3-D global simulations of mean flows and dynamo action in stars

With the advent of massively parallel computers it is becoming more tractable to attack the difficult problem of stellar convection and dynamo with full 3-D MHD non linear simulations.

4.1. *Differential rotation and Meridional Circulation in Stars*

Systematic studies of rotating convection in spherical shells to model solar-like stars have been undertaken over the last 10 years by several groups and codes (Brun & Toomre (2002), Ballot *et al.* (2007), Brown *et al.* (2008), Käpylä *et al.* (2011), Augustson *et al.* (2012), Gastine *et al.* (2013), and references therein). The general trend is that the Coriolis force modifies convection such as to establish a large scale differential rotation $\Omega(r,\theta)$ (Brun & Rempel (2009)). Depending on the influence of the Coriolis force, usually mesured by the turbulent Rossby number $Ro = \omega_{conv}/2\Omega_0 \sim v_{conv}/2\Omega_0 d$, or a variant, with ω_{conv}, v_{conv} characteristic vorticity and velocity in the convection zone and d the convection zone depth, the resulting differential rotation can be anti-solar (high $Ro > 1$, with fast poles-slow equator), solar-like ($0.2 < Ro < 0.9$, with fast equator, slow poles and some constancy at mid latitude of the isocontours of Ω) or Jupiter-like ($Ro < 0.1$, with cylindrical profile with alternance of prograde and retrograde jets) Matt *et al.* (2011). On Figure 3 we represent these different profiles in models for various masses (0.5, 0.7 and 1.1 Msol) that also include the coupling to a stably stratified radiative interior, thus possessing a tachocline Matt & Brun (2013). For each stellar mass these various states can be achieved but for a different effective rotation rate (or $v \sin i$). Indeed, we find that the convective velocity v_{conv} roughly scales as $(L_*/(\bar{\rho}_{cz} R_*^2))^{1/3}$. Hence, more massive is the stars, higher is its luminosity and lower is its average density as the base of the convective envelope moves outward in relative mass (the stellar radius variation for a masse range between 0.5 to 1.2 solar mass is a factor of 2 at most). The direct consequence is that the convective velocity increases significantly with stellar mass and so does the Rossby number for a fixed rotation rate. So we anticipate that the transition between prograde and retrograde differential rotation does not occur at the same rotation rate for a given spectral type. Searching for this limit observationaly would be most useful to theoreticians. We also find that the differential rotation amplitude from the equator to 60 deg increases with rotation rate but not as fast as the rotation rate such that the relative differential rotation $\Delta\Omega/\Omega_0$ reduces. 3-D numerical simulations also predict that $\Delta\Omega$ should be larger for more massive stars as shown in Figure 4 left panel. This is in qualitative agreement with observations of Donahue *et al.*(1996) and Barnes *et al.*(2005).

Likewise the meridional circulation is influenced by rotation. We generally found that for simulations with $Ro < 1$, meridional circulations possess many cells in radius and/or latitude per hemisphere. Only for anti-solar differential rotation cases, is the meridional circulation uni-cellular Matt *et al.* (2011). We also find that the amplitude decreases as the rotational influence is increased as shown on Figure 4 right panel. This comes about by having more energy diverted toward differential rotation kinetic energy reservoir as motions tends to become more horizontal than to the meridional circulation kinetic energy reservoir that requires motions to go across surfaces aligned with the rotation axis which becomes harder as Ω_0 increases. As we have seen in §3, both multi-cellular

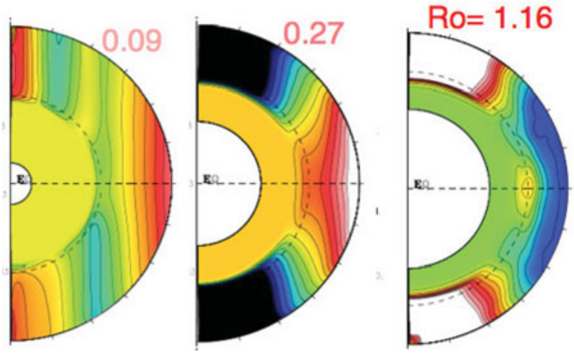

Figure 3. Differential rotation profiles realized in 3-D ASH simulations of solar-like G and K stars, chosen such as to emphasize the 3 rotation regimes as the Rossby number changes from < 0.1 to > 1: banded-cylindrical, solar-like-conical, anti-solar (slow equator-fast poles).

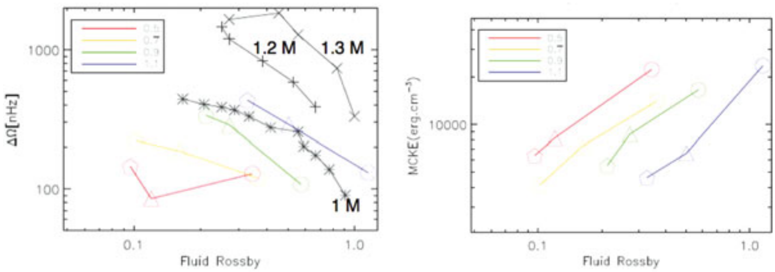

Figure 4. Left: Latitudinal variation of angular velocity contrast with Rossby number. We note that the contrast increases with faster rotation and that more massive is the star higher for a given Rossby number is its differential rotation. Right: Variation of kinetic energy in meridional circulation flow with Rossby number. We note that as the star rotates faster less energy is channeled to the meridional circualtion (Matt *et al.* (2011), Matt & Brun (2013))

meridional circulations and weaker flow amplitudes imply that we must reconsider the standard dynamo model for solar-like stars if we want to reproduce observations.

Up to now we have discussed results that did not take into account the retroaction of the magnetic field. This is of course correct as long as the feedback of the Lorentz force on motions is negligible. We now discuss in which conditions this is or not the case.

4.2. *Stellar nonlinear dynamo action, scaling laws of magnetic energy and spot-dynamo*

3-D numerical simulations of dynamo action in solar-like stars have revealed a large range of behavior, from steady dynamo, to irregular and cyclic ones (Brun *et al.* (2004), Brown *et al.* (2010), Brown *et al.* (2011), Racine *et al.*(2011), Gastine *et al.*(2012), Augustson *et al.* (2013), Käpylä *et al.*(2013), Nelson *et al.* (2013), and references therein). In particular in model with a dominant influence of the rotation, large scale magnetic wreaths (see Figure 6 left panel) have been obtained without requiring the presence of a tachocline Brown *et al.* (2010), Brown *et al.* (2011). We show on Figure 5, 4 butterfly diagrams (time-latitude plots of the azimuthally averaged toroidal magnetic field near the base of the CZ) realized in such simulations for of a solar-like star rotating at 3 times the solar rate Nelson *et al.* (2013). We remark that as the model is made more turbulent, the steady magnetic wreaths become more time dependent and can lead to cyclic activity (bottom right panel).

In such stars, along with the degree of turbulence, rotation plays an important role in determining the global properties of their magnetism. This is due to a shift in the

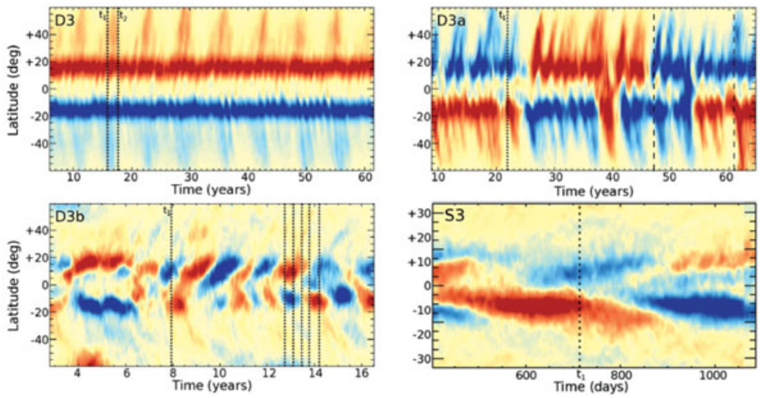

Figure 5. Magnetic wreaths yielding in turn steady (D3), irregular (D3a) and quasi cyclic (D3b & S3) magnetic butterfly diagrams (Brown *et al.* (2010), Nelson *et al.* (2013))

balance of forces driving the flow between the advection, Coriolis and Lorentz terms. As the rotation rate increases the Lorentz force tends to balance the Coriolis force yielding larger magnetic energy in superequipartion with the kinetic energy of the flow (a direct consequence of a magnetostrophic state; c.f. strong scaling below) as in the Earth's iron core. The Elsasser number $\Lambda = B^2/4\pi\bar{\rho}_{cz}\eta\Omega_0$, with $\bar{\rho}_{cz}$ mean density in the convective envelope, η magnetic diffusivity, Ω_0 stellar rotation rate, B a characteristic magnetic field of the CZ, is useful to discuss this balance of terms in the Navier-Stokes (N. V.) equation. Depending on the amplitude of this number and on the balance assumed in the Navier-Stokes equation, various scaling of the magnetic field amplitude can be expected (Fauve & Pétrélis(2007), Christensen(2010)):

• First, let's recall that an order of magnitude of an equilibrium magnetic field (assuming ideal gas law) can easily be obtained: $B_{eq} \sim \sqrt{8\pi P_{gas}} \sim \sqrt{\bar{\rho}_{cz}}$, since T_{eff} varies by a factor 2 to 3 between early F and late K stars, whereas $\bar{\rho}_{cz}$ varies by more than a factor 100 (Matt *et al.* (2011)). Now let's assume that the magnetic Reynolds number $\sim$1 such that a characteristic velocity is given by $v \sim \eta/d$, and let's study the balance of terms in N. V. eq.:

- Laminar (weak) scaling: Lorentz $\sim$ viscous diffusion
  $\Rightarrow B^2_{weak} \sim \bar{\rho}_{cz}\nu v/d \sim \bar{\rho}_{cz}\nu\eta/d^2$
- Turbulent (equipartition) scaling: Lorentz $\sim$ advection
  $\Rightarrow B^2_{turb} \sim \bar{\rho}_{cz}v^2 \sim \bar{\rho}_{cz}\eta^2/d^2 \Leftrightarrow |B_{weak}| \sim |B_{turb}|P_m^{1/2}$
- Magnetostrophic (strong) scaling (e.g. Elsasser nb $\Lambda \sim 1$): Lorentz $\sim$ Coriolis
  $\Rightarrow B^2_{strong} \sim \bar{\rho}_{cz}\Omega_0\eta$

with v, d characteristic velocity and length scales, $P_m = \nu/\eta$ the magnetic Prandtl nb. Of course there is an upper limit to the magnitude of the magnetic energy ultimately set by the amount of energy (likely the star's outward energy flux) than can be made available to the dynamo process. We recall here that dynamo action does not exist for any class of motions due to its intrinsic 3-D character (Moffatt(1978)).

So a possible scenario is the following: Stars rotating at moderate rate (such that their Elsasser number is small), have a level of magnetic energy (or averaged global field strenght) that are less than or of the order of the equipartition field given by either the weak or turbulent scalings. As stars rotate faster and get closer to be in a magnetostrophic state with an Elssasser number of order 1 or larger, the formation of large and intense magnetic wreaths starts. The magnetic field is more an more dominated by

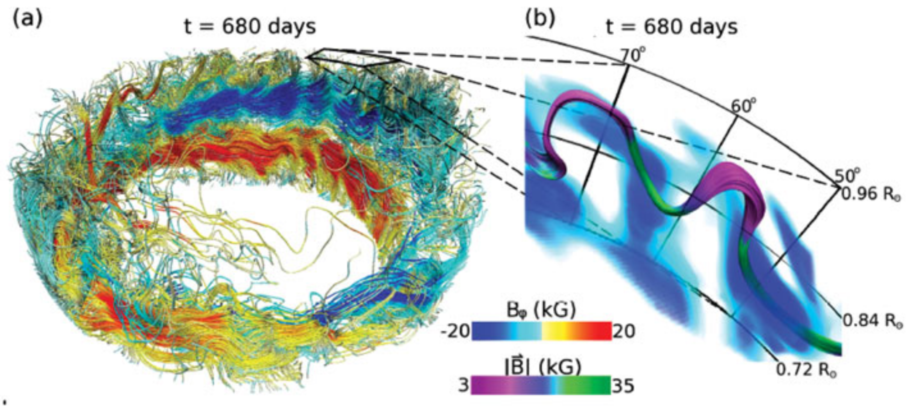

Figure 6. Turbulent magnetic wreaths (panel a) leading to the generation of buoyant loops (panel b) in case S3 (Nelson *et al.* (2013))

its toroidal component and the magnetic energy becomes larger and larger going above the equipartition value and follows the strong scaling. The consequence is the following. As the magnetic energy (or field amplitude) becomes large, the associated Lorentz force starts back reacting strongly on the mean flow. The first consequence is what can be called an "omega-quenching", e.g. the differential rotation reduces in strength and an almost solid body rotation state in the convective zone (envelope or core) is established (Brun (2004), Brun *et al.*(2005)). In mean-field classification this means that the stellar dynamo transits from being an $\alpha - \omega$ or $\alpha^2 - \omega$ to being an α^2 dynamo, i.e. helical turbulence is solely responsible for field generation and maintenance, the large scale shear now plays a marginal role. At that stage what remains of the magnetic wreaths is still unclear, more work must be done. As the rotation is made even faster, quenching of the α effect, due to the large scale magnetic field being more and more intense, occurs. The link to the L_x saturation (Pizzolato *et al.* (2003), Wright *et al.* (2011)) is not as straightforward to deduce as one must also assess how the filling factor of the magnetic field on the star's surface evolves with stellar parameter not just the field strenght. As we have seen the field amplitude does not easily saturate. So one can supposely think that the first saturation is due to ω-quenching and limitation of overall spot coverage and the second "over-saturation" may be due to "α-quenching", so of the actual field strength (see Gondoin(2012) for an alternative explanation). In order to be able to properly set the transition and the saturation of B and f independently, *spot-dynamo*, e.g. dynamo generating self consitently rising omega-loop must be developped and the parameter space explored systematically. We show a first step toward that goal in Figure 6 where we see a magnetic wreath-like structure becoming turbulent and intermittent enough, that intense bundles of fields reach 50 kG and start becoming buoyant, forming omega-loop like structures (Nelson *et al.* (2011), Nelson *et al.* (2013)). We believe that such simulations are the progenitor of future more realistic *spot-dynamos*.

5. conclusion

By extending the concept of dynamo to other stellar spectral type ($F \rightarrow M$), it would seem that a transition occurs from $\alpha - \omega$ + flux transport dynamos, to α^2 dynamos and a turbulent dynamo. We have seen that theory and numerical simulations can explain qualitatively the general trends. We now summarize the most important results:

• Convective velocities v_{conv} roughly scales with cubic root of $L_*/(\bar{\rho}_{cz} R_*^2)$ (stars luminosity devided by mean density in CZ and stellar radius squared). So it implies that prograde vs retrograde state changes at different Ω_0 as spectral type is changed (since $Ro = v_{conv}/(2\Omega_0 d)$ and v_{conv} changes with spectral type),

• Cylindrical vs conical vs shellular differential profiles depends on Reynolds stresses and thermal (baroclinic) effects (see Miesch *et al.* (2006) and Ballot *et al.* (2007) for more details). Larger absolute differential rotation for both more massive stars and higher rotation rates are recovered,

• The meridional circulation is found to be weaker for faster rotation rate, due to relatively more energy being channeled to longitudinal motions,

• Magnetic field B reduces or can even supress differential rotation $\Omega(r, \theta)$ (ω-quenching),

• at high rotation rate we get magnetic wreaths that generate omega-loops as we lower diffusivity, cyclic dynamos are easier to get, and a new concept of *spot-dynamo* has emerged,

• Strength of field (weak/strong) depends on balance of forces in N.V eq. and Multipolar or Dipolar magnetic bi-stability can exist but multipolar fields seem to dominate at high stratification,

• Observed stellar cycle period becomes shorter for faster rotation, implies to modify the standard flux transport mean field dynamo model to include either multi-cellular flow or turbulent pumping,

• Stratification and/or a tachocline and/or a low P_m may help getting equatorward butterfly diagram (results in a shift of location of $\Omega(r, \theta)$ and α-like effects and hence of their phase relationship).

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References

Augustson, K. C., Brown, B. P., Brun, A. S., Miesch, M. S., & Toomre, J. 2012, *ApJ*, 756, 169
Augustson, K. C., Brun, A. S., & Toomre, J. 2013, *ApJ*, 777, 153
Babcock, H. W. 1961, *ApJ*, 133, 572
Baliunas, S. L., Donahue, R. A., Soon, W. H., *et al.* 1995, *ApJ*, 438, 269
Baliunas, S. L., Nesme-Ribes, E., Sokoloff, D., & Soon, W. H. 1996, *ApJ*, 460, 848
Ballot, J., Brun, A. S., & Turck-Chièze, S. 2007, *ApJ*, 669, 1190
Barnes, J. R., Collier Cameron, A., Donati, J.-F., *et al.* 2005, *MNRAS*, 357, L1
Böhm-Vitense, E. 2007, *ApJ*, 657, 486
Brown, B. P., Browning, M. K., Brun, A. S., Miesch, M. S., & Toomre, J. 2008, *ApJ*, 689, 1354
Brown, B. P., Browning, M. K., Brun, A. S., Miesch, M. S., & Toomre, J. 2010, *ApJ*, 711, 424
Brown, B. P., Miesch, M. S., Browning, M. K., Brun, A. S., & Toomre, J. 2011, *ApJ*, 731, 69
Browning, M. K., Miesch, M. S., Brun, A. S., & Toomre, J. 2006, *ApJL*, 648, L157
Brun, A. S. 2004, *Solar Phys.*, 220, 333
Brun, A. S. & Toomre, J. 2002, *ApJ*, 570, 865
Brun, A. S., Miesch, M. S., & Toomre, J. 2004, *ApJ*, 614, 1073
Brun, A. S., Browning, M. K., & Toomre, J. 2005, *ApJ*, 629, 461
Brun, A. S. & Rempel, M. 2009, *Space Sci. Rev.*, 144, 151
Brun, A. S., Miesch, M. S., & Toomre, J. 2011, *ApJ*, 742, 79
Charbonneau, P. & Saar, S. H. 2001, Magnetic Fields Across the H-R Diagram, 248, 189
Charbonneau, P. 2010, Living Reviews in Solar Physics, 7, 3
Christensen, U. R. 2010, *Space Sci. Rev.*, 152, 565
D'Silva, S. & Choudhuri, A. R. 1993, *A&A*, 272, 621
DeRosa, M. L., Brun, A. S., & Hoeksema, J. T. 2012, *ApJ*, 757, 96

Dikpati, M., Saar, S. H., Brummell, N., & Charbonneau, P. 2001, Magnetic Fields Across the H-R Diagram, 248, 235
Dikpati, M., de Toma, G., Gilman, P. A., Arge, C. N., & White, O. R. 2004, *ApJ*, 601, 1136
Do Cao, O. & Brun, A. S. 2011, Astron. Nach., 332, 907
Donahue, R. A., Saar, S. H., & Baliunas, S. L. 1996, *ApJ*, 466, 384
Donati, J.-F., Semel, M., Carter, B., Rees, D., & Collier Cameron, A. 1997, *MNRAS*, 291, 658
Dubé, C. & Charbonneau, P. 2013, *ApJ*, 775, 69
Durney, B. R. & Latour, J. 1978, Geophysical and Astrophysical Fluid Dynamics, 9, 241
Fauve, S. & Pétrélis, F. 2007, Comptes Rendus Physique, 8, 87
García, R. A., Mathur, S., Salabert, D., *et al.* 2010, Science, 329, 1032
Gastine, T., Duarte, L., & Wicht, J. 2012, *A&A*, 546, A19
Gastine, T., Yadav, R., Morin, J *et al.* 2013, *MNRAS*, in press
Giampapa, M. S. 2005, Saas-Fee Adv. Course 34: The Sun, Solar Analogs and the Climate, 307
Giampapa, M. S., Hall, J. C., Radick, R. R., & Baliunas, S. L. 2006, *ApJ*, 651, 444
Gondoin, P. 2012, *A&A*, 546, A117
Guerrero, G. & de Gouveia Dal Pino, E. M. 2008, *A&A*, 485, 267
Hazra, G., Karak, B. B., & Choudhuri, A. R. 2013, arXiv:1309.2838
Jouve, L. & Brun, A. S. 2007, *A&A*, 474, 239
Jouve, L., Brown, B. P., & Brun, A. S. 2010, *A&A*, 509, A32
Jouve, L., Brun, A. S., & Aulanier, G. 2013, *ApJ*, 762, 4
Käpylä, P. J., Korpi, M. J., Ossendrijver, M., & Stix, M. 2006, *A&A*, 455, 401
Käpylä, P. J., Mantere, M. J., Guerrero, G., Brandenburg, A., & Chatterjee, P. 2011, *A&A*, 531, A162
Käpylä, P. J., Mantere, M. J., Cole, E., Warnecke, J., & Brandenburg, A. 2013, *ApJ*, 778, 41
Küker, M., Rüdiger, G., & Kitchatinov, L. L. 2011, *A&A*, 530, A48
Leighton, R. B. 1969, *ApJ*, 156, 1
Mathur, S., García, R. A., Morgenthaler, A., *et al.* 2013, *A&A*, 550, A32
Matt, S. P., Do Cao, O., Brown, B. P., & Brun, A. S. 2011, Astron. Nach., 332, 897
Matt, S. P. & Brun, A. S. 2013, ApJ submitted
Miesch, M. S., Brun, A. S., & Toomre, J. 2006, *ApJ*, 641, 618
Miesch, M. S., Brun, A. S., De Rosa, M. L., & Toomre, J. 2008, *ApJ*, 673, 557
Moffatt, H. K. 1978, Cambridge, England, Cambridge University Press, 1978. 353 p.
Montesinos, B., Thomas, J. H., Ventura, P., & Mazzitelli, I. 2001, *MNRAS*, 326, 877
Morgenthaler, A., Petit, P., Morin, J., *et al.* 2011, Astronomische Nachrichten, 332, 866
Nandy, D. 2004, *Solar Phys.*, 224, 161
Nandy, D. & Martens, P. C. H. 2007, Advances in Space Research, 40, 891
Nelson, N. J., Brown, B. P., Brun, A. S., Miesch, M. S., & Toomre, J. 2011, *ApJL*, 739, L38
Nelson, N. J., Brown, B. P., Brun, A. S., Miesch, M. S., & Toomre, J. 2013, *ApJ*, 762, 73
Noyes, R. W., Weiss, N. O., & Vaughan, A. H. 1984, *ApJ*, 287, 769
Petit, P., Dintrans, B., Solanki, S. K., *et al.* 2008, *MNRAS*, 388, 80
Pizzolato, N., Maggio, A., Micela, G., Sciortino, S., & Ventura, P. 2003, *A&A*, 397, 147
Pouquet, A., Frisch, U., & Leorat, J. 1976, Journal of Fluid Mechanics, 77, 321
Racine, É., Charbonneau, P., Ghizaru, M., Bouchat, A., & Smolarkiewicz, P. 2011, *ApJ*, 735, 46
Reiners, A. 2012, Living Reviews in Solar Physics, 9, 1
Saar, S. 2002, Stellar Coronae in the Chandra and XMM-NEWTON Era, 277, 311
Saar, S. H. & Brandenburg, A. 1999, *ApJ*, 524, 295
Strugarek, A., Brun, A. S., Mathis, S., & Sarazin, Y. 2013, *ApJ*, 764, 189
Tobias, S. M. 1997, *A&A*, 322, 1007
Tobias, S. M., Brummell, N. H., Clune, T. L., & Toomre, J. 2001, *ApJ*, 549, 1183
Weiss, N. O. 1994, Lectures on Solar and Planetary Dynamos, 59
Wilson, O. C. 1978, *ApJ*, 226, 379
Wright, N. J., Drake, J. J., Mamajek, E. E., & Henry, G. W. 2011, *ApJ*, 743, 48
Zhao, J., Bogart, R., Kosovichev, A., Duvall, T., Jr., & Hartlep, T. 2013, *ApJL*, 774, L29