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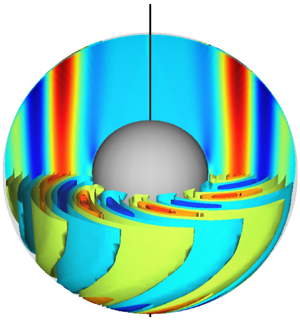

Convection-driven dynamos in rotating spherical shells - basic phenomenology

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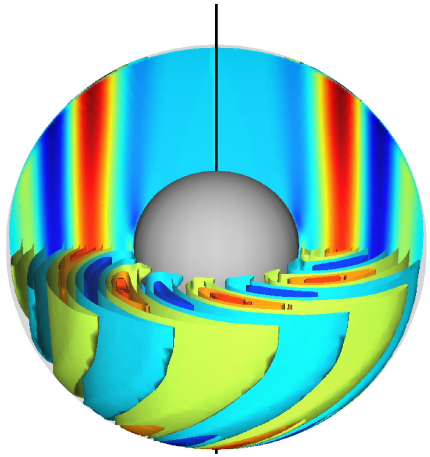
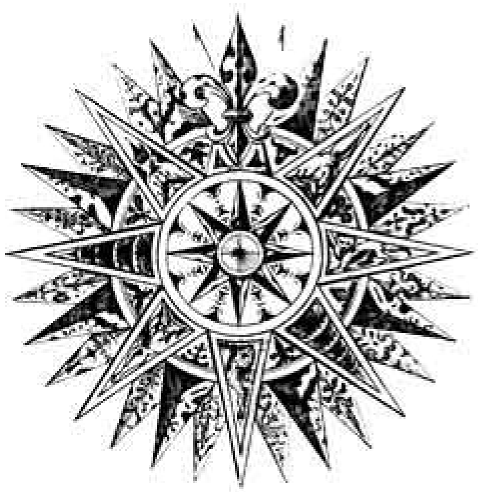
East Kilbride, 28 Oct 2011

Motivation: Applications of spherical dynamos

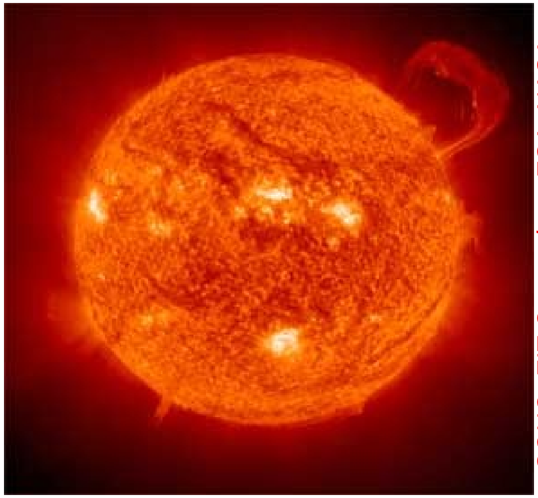


Dynamos in rotating spherical shells



Geomagnetism

Planetary magnetism



SOHO, EIT Consortium, ESA, NASA

Solar and stellar magnetism

Outline of the talk

- *Mathematical formulation of the problem.*
- *Numerical methods of solution.*
- *Typical convection and dynamo features. Turbulence.*
- *Overview of the basic effects controlling dynamo behaviour and types of dynamo solutions.*
- *Oscillations of dipolar dynamos as a possible cause of geomagnetic excursions and reversals.*
- *Bistability and hysteresis of fully nonlinear dypolar dynamos.*
- *Conclusions.*

Model remarks

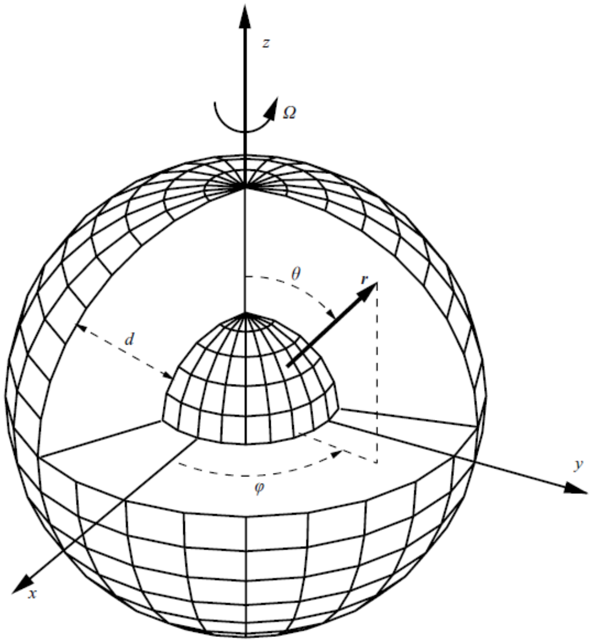
*Our main motivation has been to model **planetary dynamos and the Geodynamo.** Accordingly, we have made a number of appropriate assumptions:*

- ***Boussinesq approximation** – constant material properties; variation of density is only included in the gravity term.*
- ***Incompressible fluid** – the velocity field is solenoidal.*
- ***Rapid rotation** – we strive to increase the rotation rate as our main motivation has been to model the Geodynamo.*
- ***Relatively thick spherical shells** - our main motivation has been to model the Geodynamo.*
- ***Direct numerical simulations** – no assumptions for the eddy diffusivities.*
- ***Self-sustained magnetic field** – we look for dynamo solutions and we do not impose external magnetic fields.*

*These may not always be appropriate in the Solar context but we believe they **capture the basic physics of the dynamo process.***

***OUR APPROACH:** systematic study of parameter dependencies and careful extrapolation to astrophyiscal objects – Earth, planets, stars.*

Convective spherical shell dynamos



Model equations & parameters

Boussinesq approximation

$$\nabla \cdot \boldsymbol{u} = 0, \quad \nabla \cdot \boldsymbol{B} = 0,$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} =$$
$$-\nabla \pi - \tau \boldsymbol{k} \times \boldsymbol{u} + \Theta \boldsymbol{r} + \nabla^2 \boldsymbol{u} + \boldsymbol{B} \cdot \nabla \boldsymbol{B},$$

$$P\left(\partial_t \Theta + \boldsymbol{u} \cdot \nabla \Theta\right) = R\, \boldsymbol{r} \cdot \boldsymbol{u} + \nabla^2 \Theta,$$

$$P_m(\partial_t \boldsymbol{B} + \boldsymbol{u} \cdot \nabla \boldsymbol{B}) = P_m\, \boldsymbol{B} \cdot \nabla \boldsymbol{u} + \nabla^2 \boldsymbol{B}.$$

$$R = \frac{\alpha\gamma\beta d^6}{\nu\kappa}, \quad \tau = \frac{2\Omega d^2}{\nu}, \quad P = \frac{\nu}{\kappa}, \quad P_m = \frac{\nu}{\lambda}$$

Simitev & Busse, JFM, 2005

Basic state & scaling

$$T_S = T_0 - \beta d^2 r^2/2$$

$$\boldsymbol{g} = -d\gamma \boldsymbol{r}$$

Length scale: d

Time scale: d^2/ν

Temp. scale: $\nu^2/\gamma\alpha d^4$

Magn. flux density: $\nu(\mu\varrho)^{1/2}/d$

Boundary Conditions

$$\boldsymbol{r} \cdot \boldsymbol{u} = \boldsymbol{r} \cdot \nabla \boldsymbol{r} \times \boldsymbol{u}/r^2 = 0,$$

$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} \cdot \boldsymbol{B}_{\text{int}} = \hat{\boldsymbol{e}}_{\boldsymbol{r}} \cdot \boldsymbol{B}_{\text{ext}},$$

$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} \times \boldsymbol{B}_{\text{int}} = \hat{\boldsymbol{e}}_{\boldsymbol{r}} \times \boldsymbol{B}_{\text{ext}},$$

$$\Theta = 0, \text{ at } r = r_i \equiv 2/3 \text{ and } r_o \equiv 5/3$$

Numerical Methods

3D non-linear problem:

Toroidal-poloidal representation

$$\boldsymbol{u} = \nabla \times (\nabla v \times \boldsymbol{r}) + \nabla w \times \boldsymbol{r} \;, \qquad \boldsymbol{B} = \nabla \times (\nabla h \times \boldsymbol{r}) + \nabla g \times \boldsymbol{r}$$

Spectral decomposition in spherical harmonics and Chebyshev polynomials

$$x = \sum_{l,m,n} X_{l,n}^m(t)\, T_n(r)\, P_l^m(\cos\theta) e^{im\varphi} \qquad \text{where } x = (v, w, \Theta, g, h)^T$$

Scalar equations

$$\partial_t X_{l,n}^m = \hat{\mathcal{L}} X_{l,n}^m + N_{l,n}^m(X) \qquad \text{where } \hat{\mathcal{L}} X_{l,n}^m\text{: linear}, N_{l,n}^m(X)\text{: non-linear}$$

Pseudo-spectral method. Time-stepping: Crank-Nicolson & Adams-Bashforth

$$[X_{l,n}^m]^{k+1} = \left(1 - \frac{\Delta t}{2}\hat{\mathcal{L}}\right)^{-1} \left\{ \left(1 + \frac{\Delta t}{2}\hat{\mathcal{L}}\right) [F_{l,n}^m]^k + \frac{\Delta t}{2}\left(3[N_{l,n}^m]^k - [N_{l,n}^m]^{k-1}\right) \right\}$$

Resolution: *radial=49,* *latitudinal=96,* *azimuthal=193.*

Linear problem:

Galerkin spectral method for the linearised equations leading to an eigenvalue problem for the critical parameters.

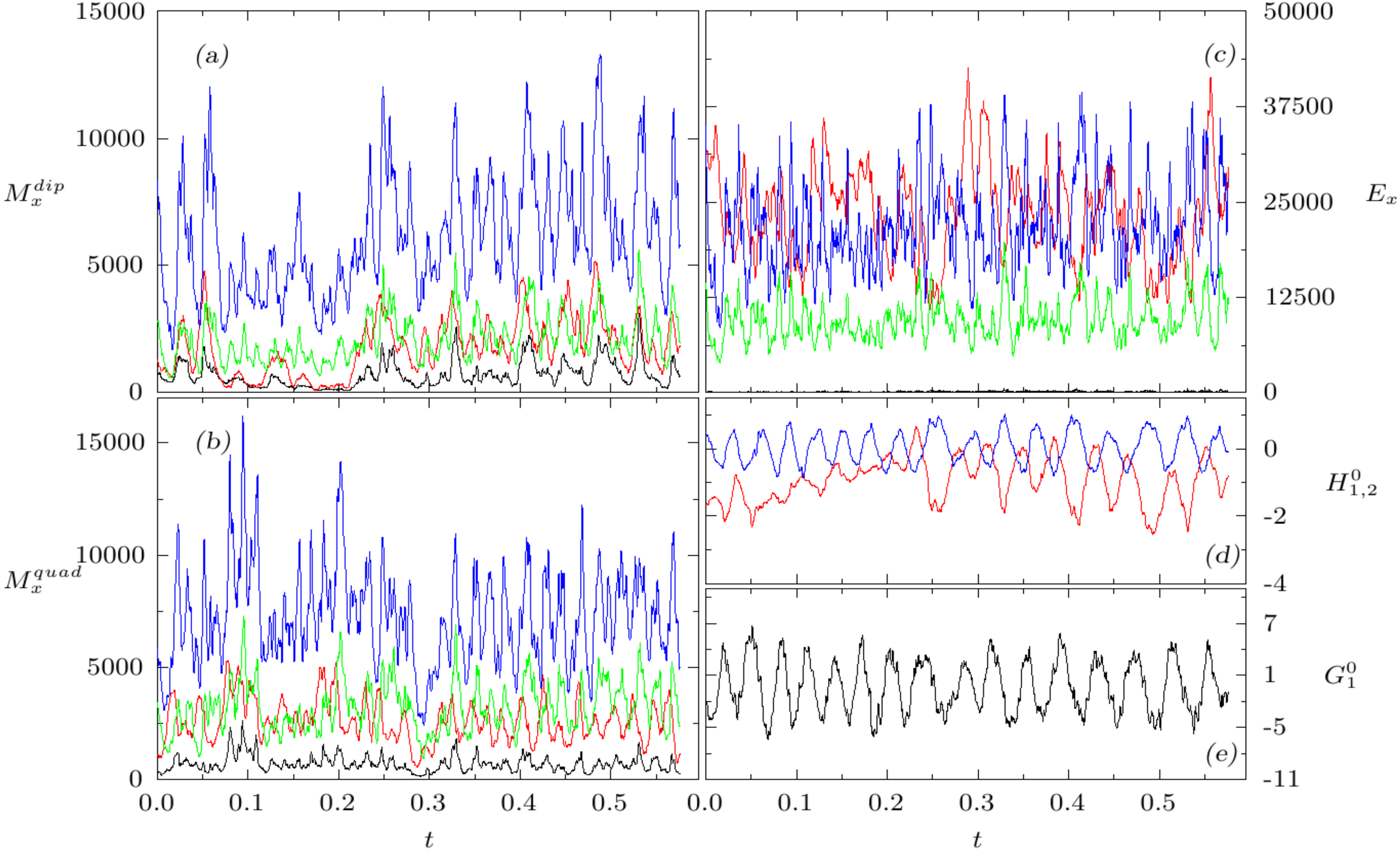
Tilgner, IJNMF, 1999

Part I

Typical properties of dynamo solutions. Turbulence

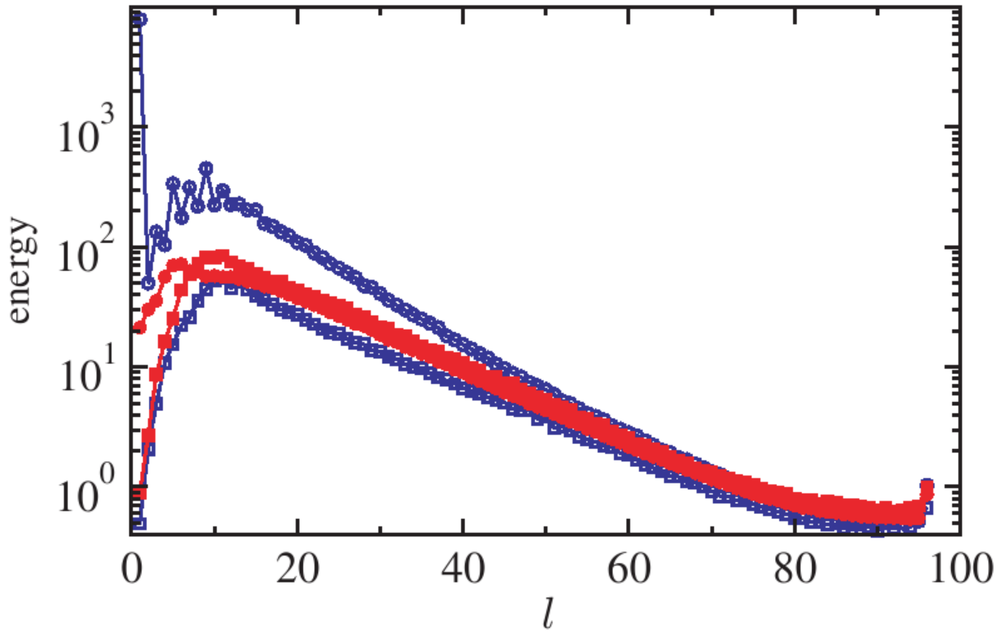
- *Spectra and spacial features*
- *Temporal behaviour*
- *Values of the magnetic Reynolds number*
- *States of convection*
- *Dynamo symmetries*

Typical time dependence: turbulence



$$P = 0.1,\ \tau = 3 \times 10^4,\ R = 1.2 \times 10^6,\ Pm = 0.5$$

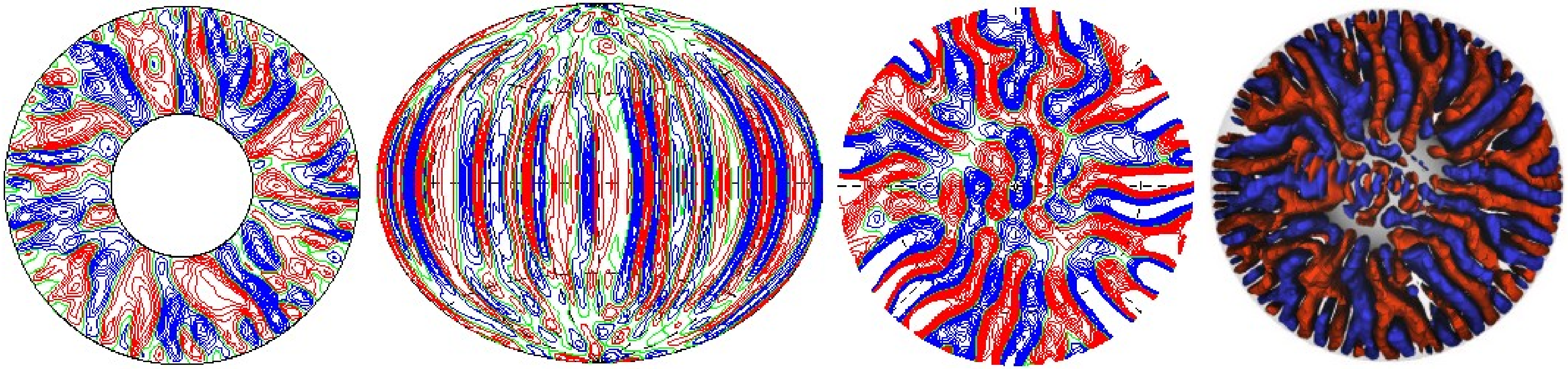
Spectra and separation of scales



$$P = 0.75, \tau = 3 \times 10^4$$

$$R = 3.5 \times 10^6, P_m = 0.75$$

MD – red, FD - blue



MD

Typical magnetic Reynolds number for dynamo onset

$Rm \;= (2E)^{1/2} P_m$

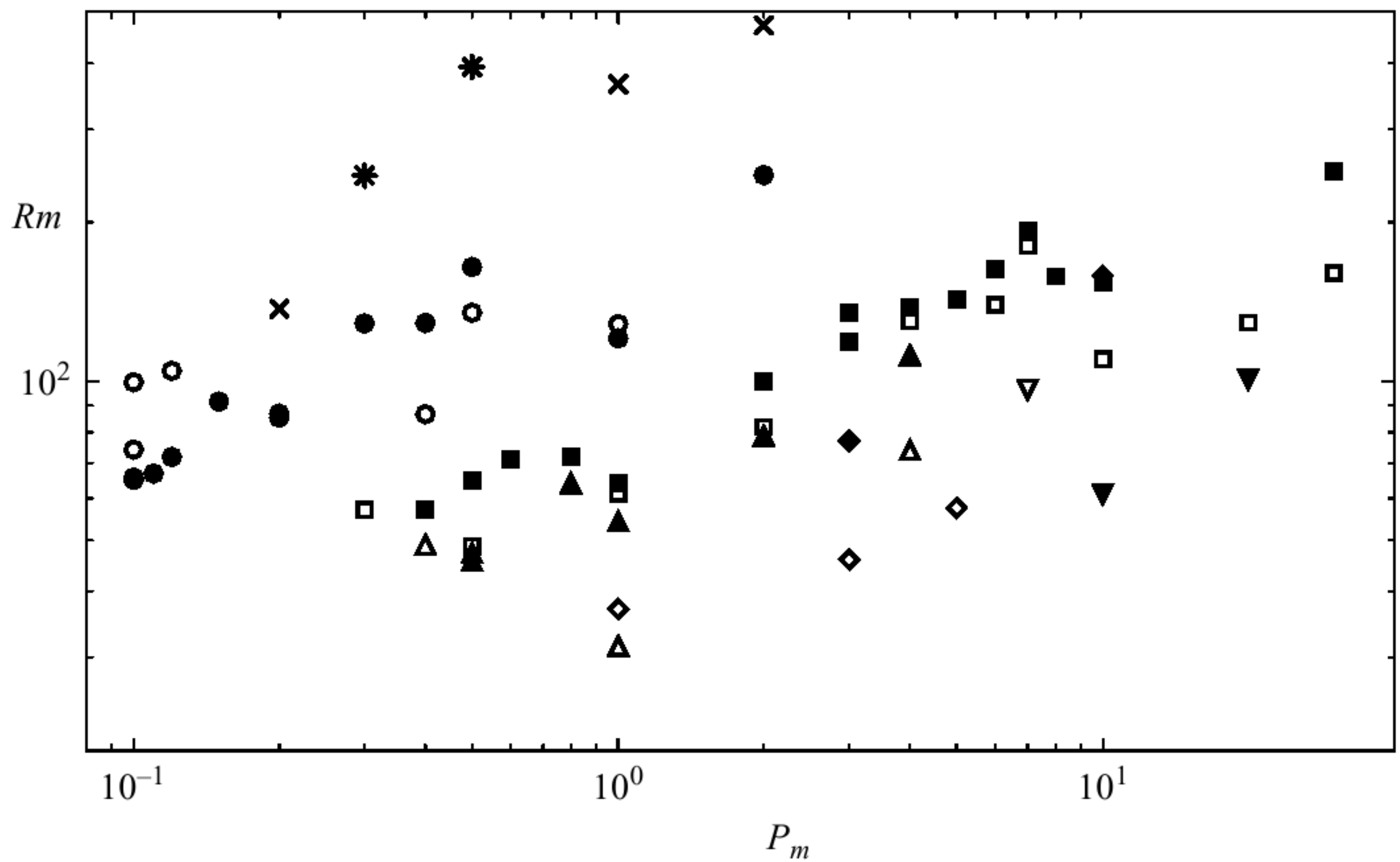
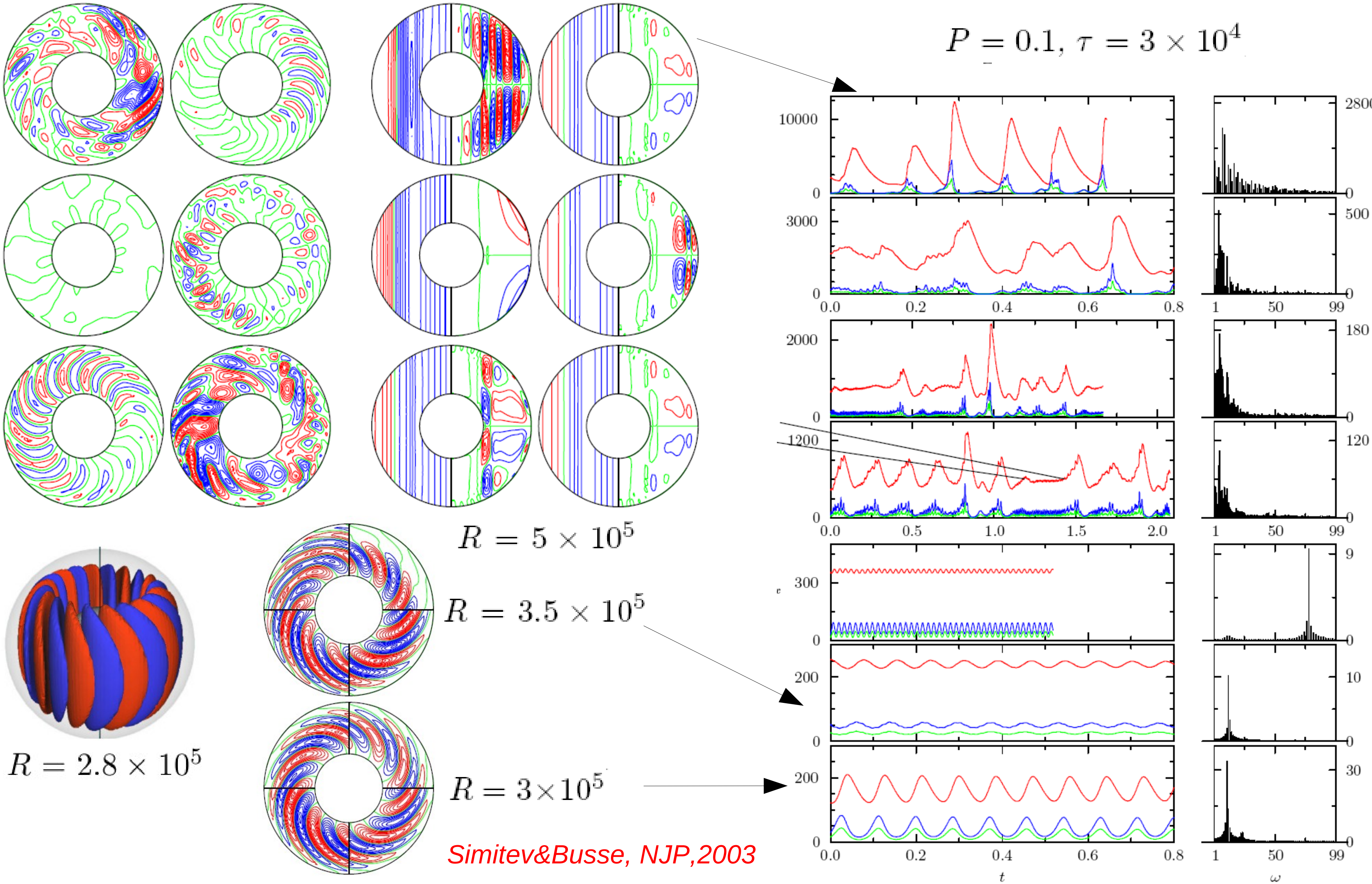


FIGURE 7. Magnetic Reynolds numbers Rm for the onset of dynamo action as a function of P_m in the cases $P = 0.01$, $\tau = 10^5$ (stars), $P = 0.025$, $\tau = 10^5$ (crosses), $P = 0.1$, $\tau = 10^5$ (circles), $P = 1$, $\tau = 3 \times 10^4$ (triangles up), $P = 1$, $\tau = 10^4$ (squares) and $P = 5$, $\tau = 5 \times 10^3$ (diamonds) and $P = 10$, $\tau = 5 \times 10^3$ (triangles down). The open symbols in the cases with $P \geqslant 0.1$ are based on decaying dynamos, the filled symbols are based on the lowest non-decaying solutions.

Finite-amplitude columnar convection



Simitev&Busse, NJP,2003

Dynamo symmetry types

Dynamo solutions exhibit symmetry because rapidly-rotating convection remains equatorially-symmetric even in the turbulent regime.

Dipolar

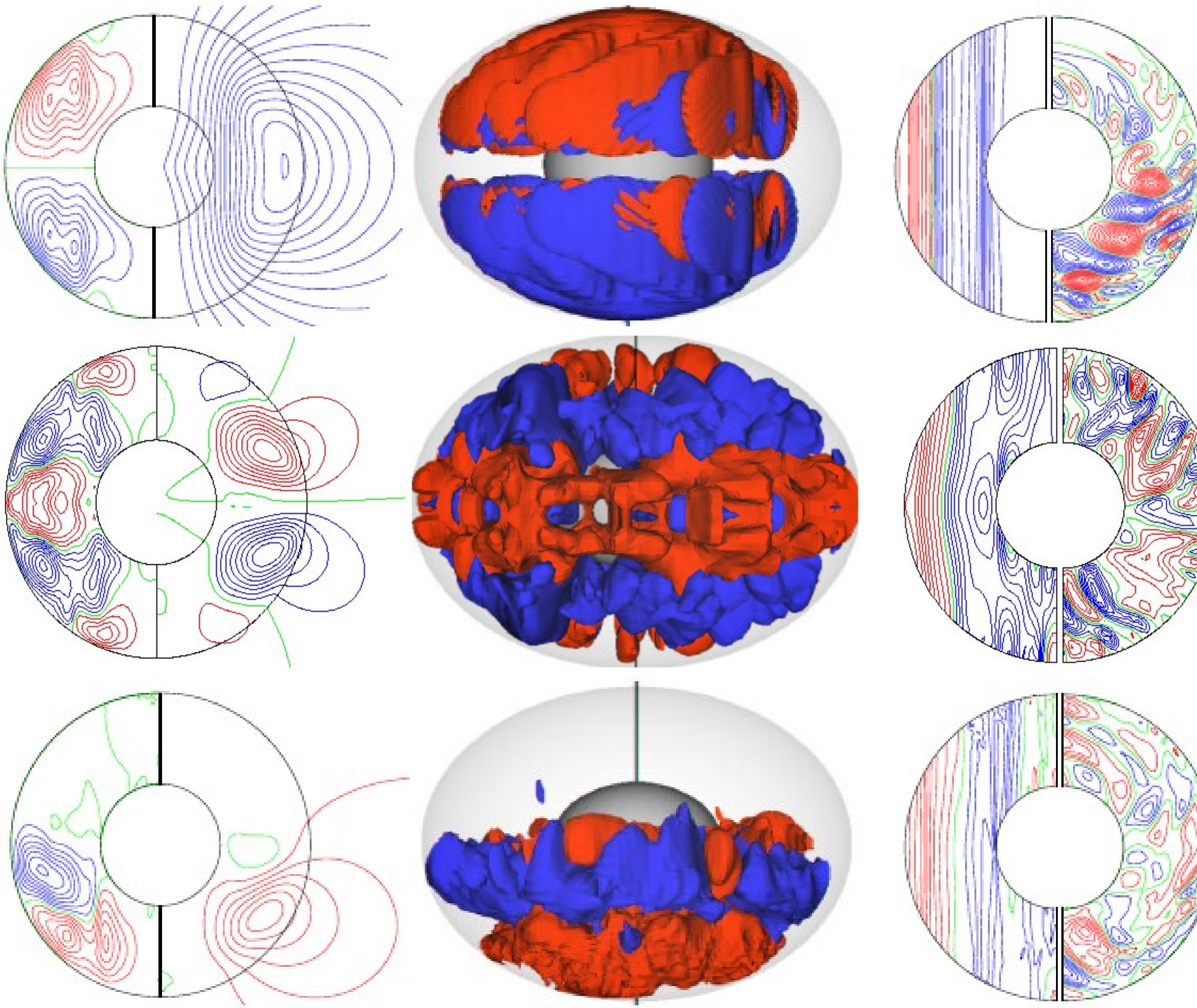
$P = 0.1,\ \tau = 10^5$
$R = 2 \times 10^6,\ Pm = 1$

Quadrupolar

$P = 5,\ \tau = 5 \times 10^3$
$R = 8 \times 10^5,\ Pm = 3$

Hemispherical

$P = 0.1,\ \tau = 10^5$
$Pm = 0.11$
$R = 6 \times 10^6$



Simitev, 2004

Part II

Basic effects controlling dynamo behaviour

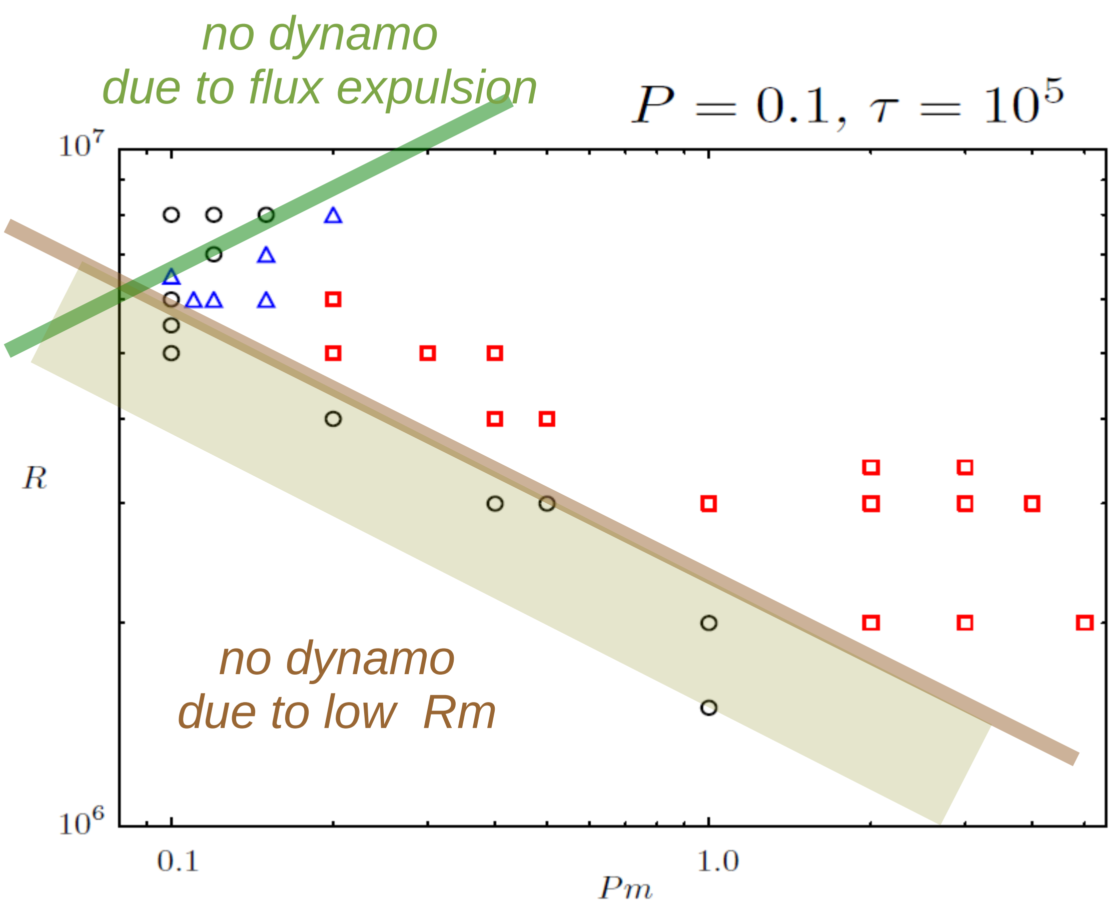
Ref: *Simitev R.D. & Busse F.H.,* ***J. Fluid Mech.****, 532, 365, 2005.*

Busse F.H. & Simitev R.D., ***Astron. Nachr****., 326, 2005.*

Grote et al., 2003.

- *Bounds on the region of dynamo action*
  - *Critical value of Rm,*
  - *Turbulent flux expulsion.*
- *Dynamo symmetry types as function of the magnetic Prandtl number.*
- *Effect of magnetic field on convection.*

Bounds on the region for dynamo action (R-Pm plane)



○ *no dynamo*

□ *chaotic dipole*

△ *hemispherical*

Simitev & Busse, JFM, 2005

Dynamo action is restricted by:

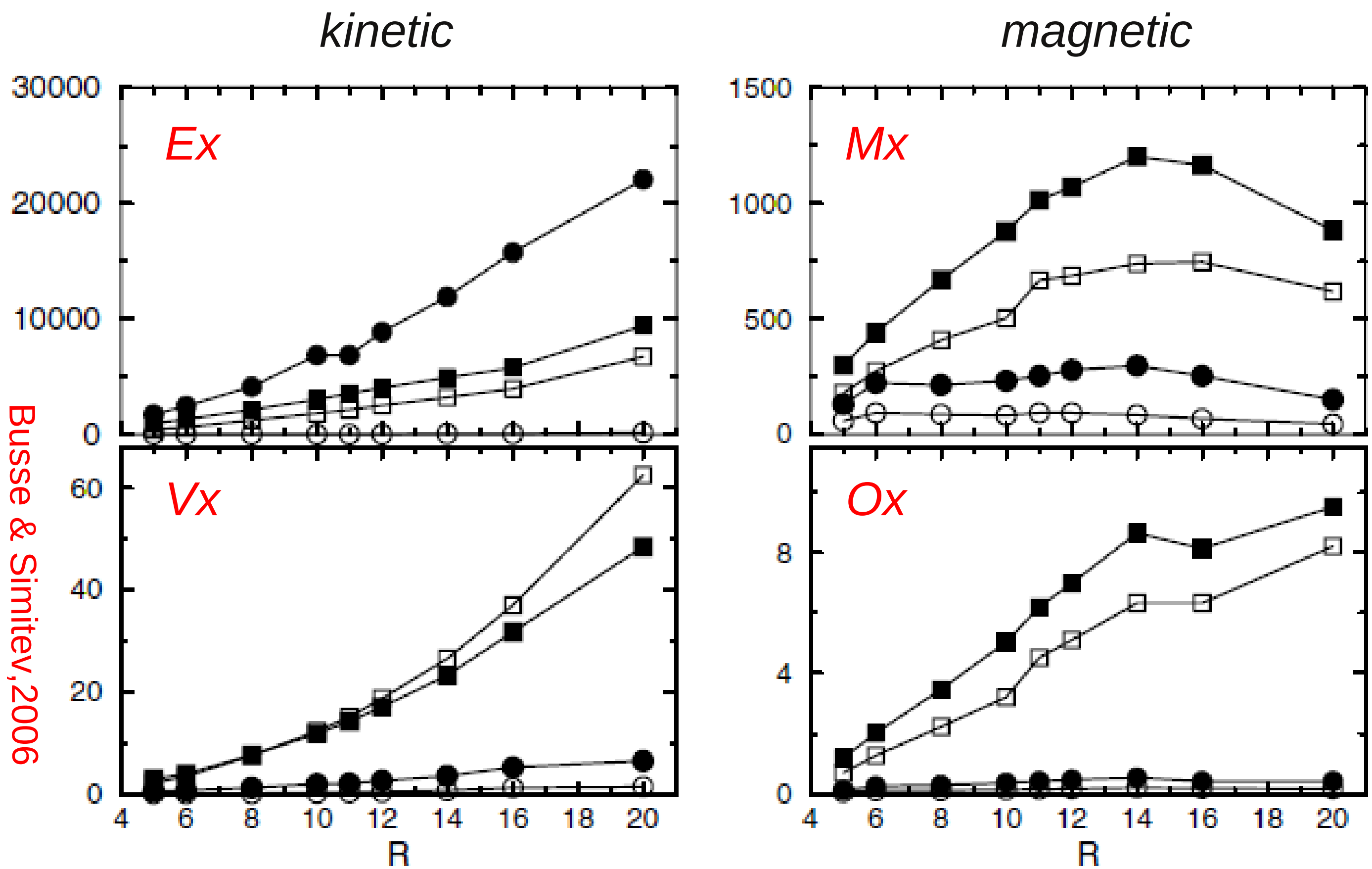
(a) ***vigour of convection*** *- convection must be sufficiently vigorous to support dynamo action.*

(b) ***magnetic field diffusivity*** *– the magnetic diffusivity must be sufficiently low for the magnetic field to persist.*

(c) ***flux expulsion*** *- however, convection which is too vigorous can lead toexpulsion of magnetic field from small eddies.*

Decay of dynamo action due to flux expulsion

(A) $\tau = 5 \times 10^3, P = P_m = 1$

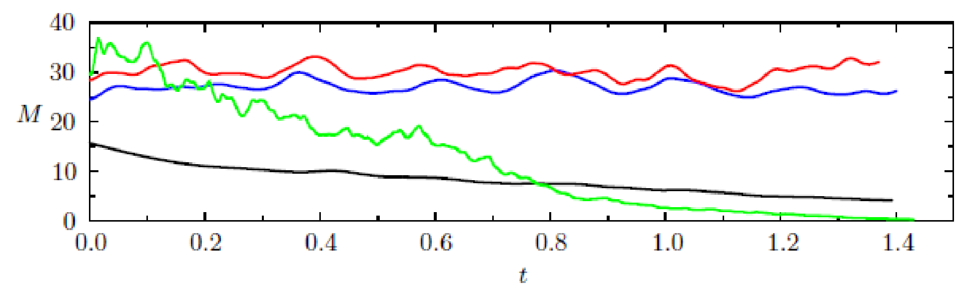


Note: With the increase of the value of Rayleigh number at all other parameter values fixed the magnetic energy components saturate and ultimately decrease due to flux expulsion and increasingly filamentary structure of the magn field.

Note: Ohmic dissipation continues to increase with R.

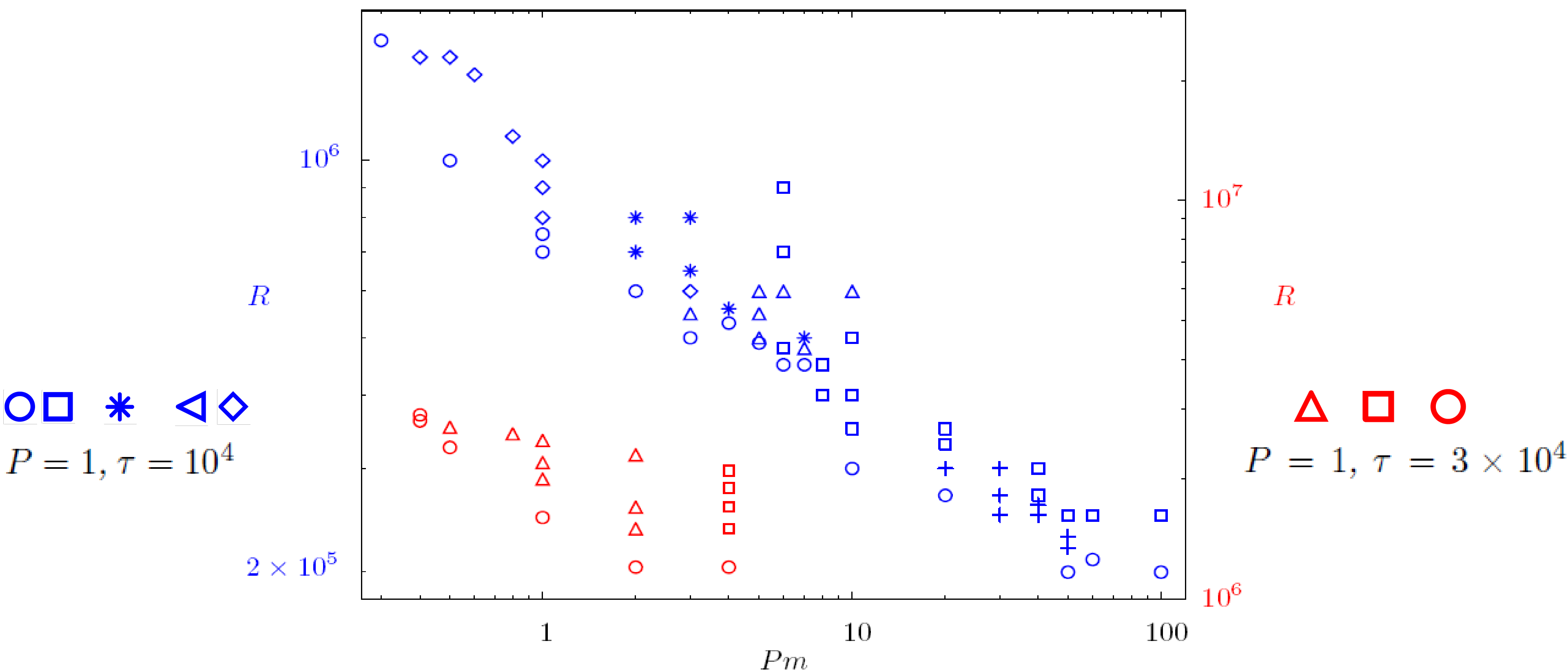
(B) $P = Pm = 10$
$\tau = 5 \times 10^5$

R=**300000 - decay**
500000 - dynamo
600000 - dynamo
700000 - decay

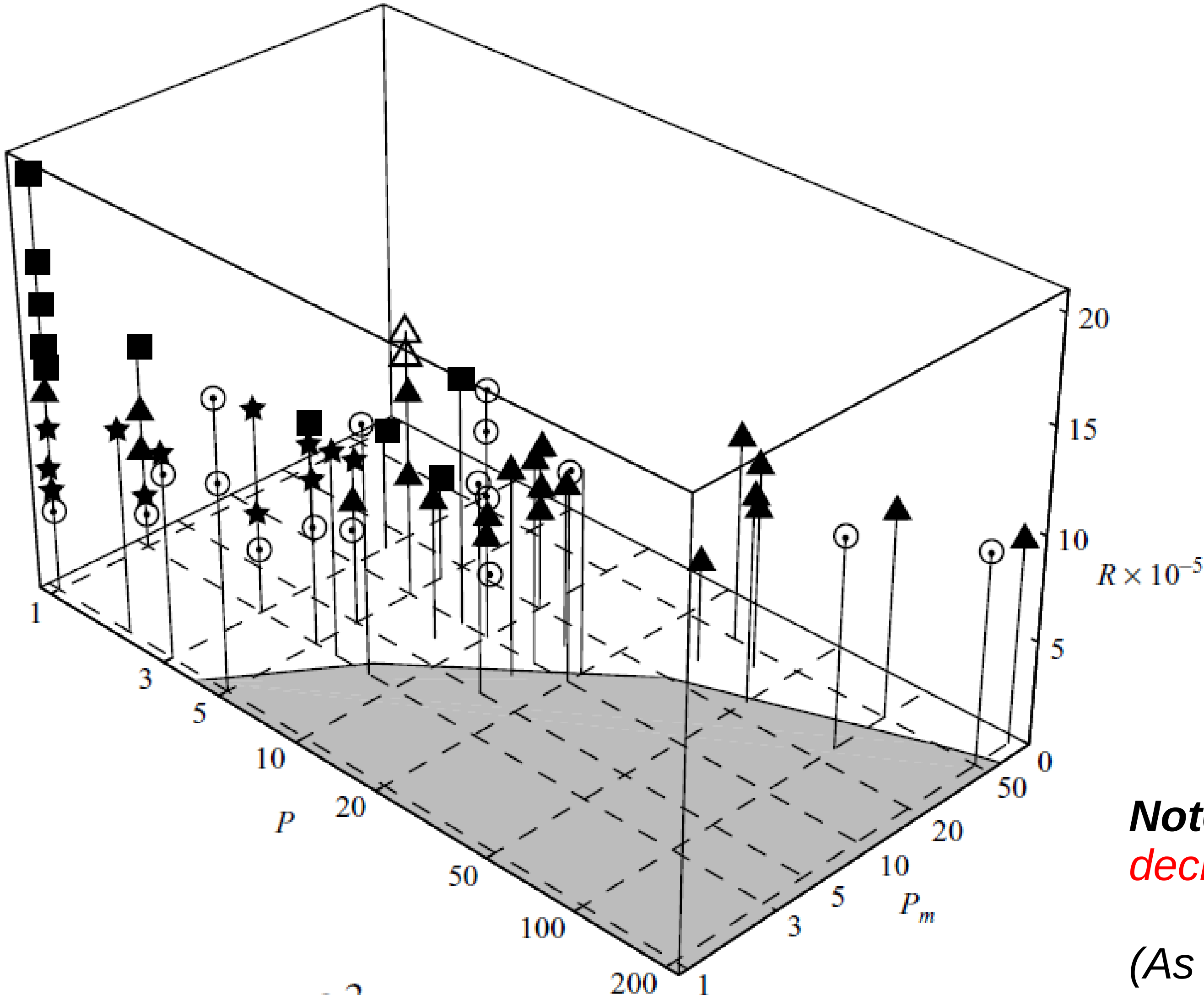


Types of dynamos in the parameter space

- No dynamo
- Regular and chaotic non-oscillatory dipolar dynamos (at large Pm/P and not far above dynamo onset)
- Oscillatory dipolar dynamos (at values of R larger than those of non-oscillatory dipoles)
- Hemispherical dynamos – always oscillatory
- Quadrupolar dynamos – always oscillatory



Bounds on the value of Pm for dynamo action (P-Pm plane)



$$\tau = 5 \times 10^3$$

(▲) *dipolar*

(△) *hemispherical*

(★) *quadrupolar*

(■) *mixed*

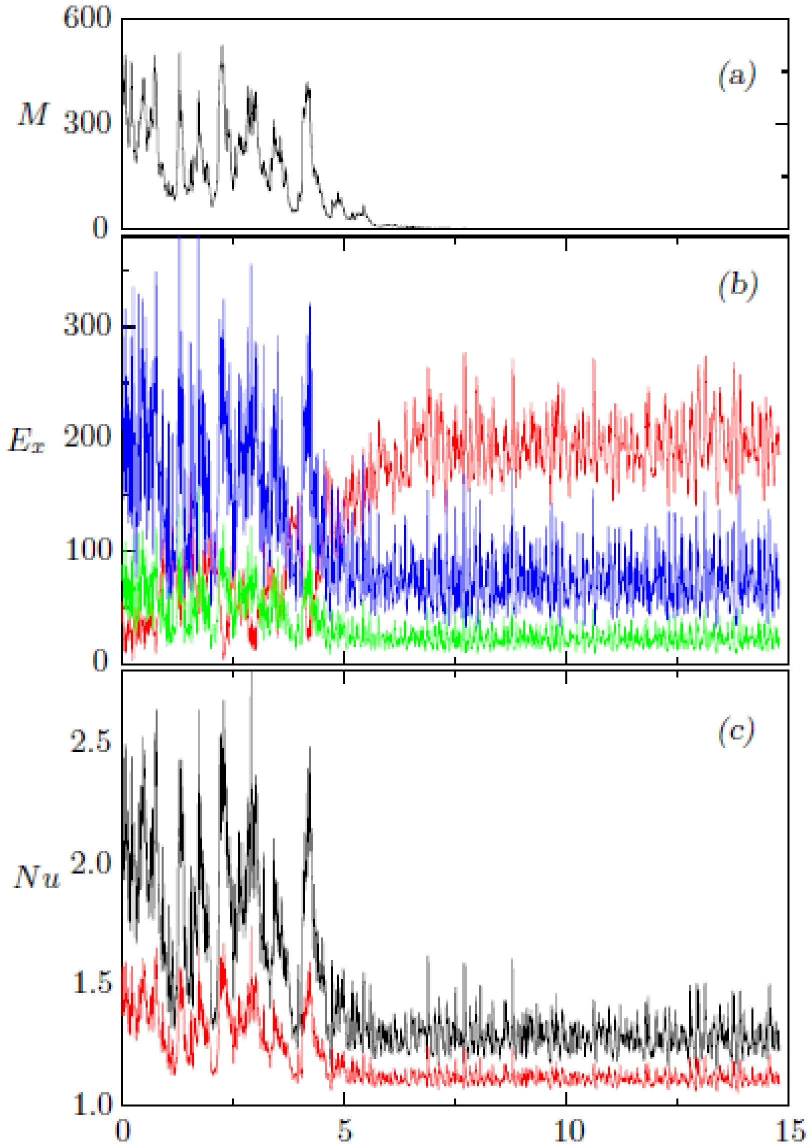
(⊙) *decaying*

Simitev & Busse, JFM, 2005

Note: *The minimal value of Pm decreases as P decreases.*

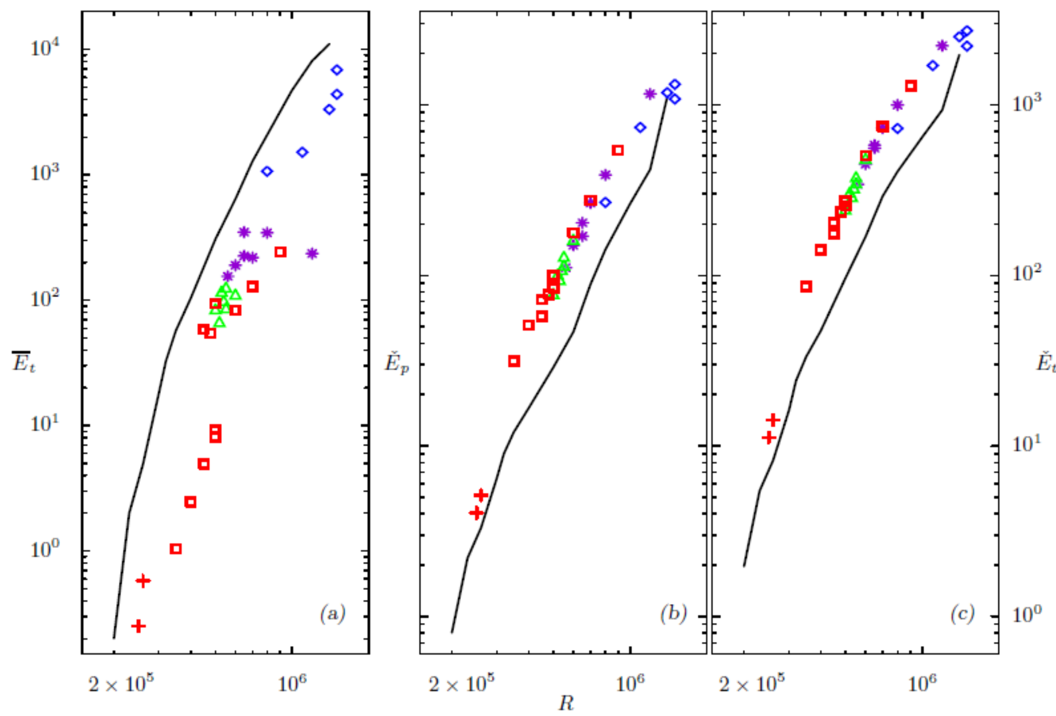
(As a rough rule the value of the critical Pm is of the same order as the ordinary Prandtl number)

Effect of self-sustained magnetic field on convection



$P = 1 \quad Pm = 7$

$\tau = 10^4, \; R = 4.5 \times 10^5$



$P = 1$ and $\tau = 10^4$

Simitev, 2004

There is little evidence that a generated magnetic field plays a role similar to externally imposed field and counteracts the Coriolis force.

Rather, the main effect of a generated field is to inhibit differental rotation and thereby increase amplitude of convection and its heat transport.

Part III

Oscillations of dipolar dynamos as a possible cause of geomagnetic excursions and reversals

Ref: *Busse F.H. & Simitev R.D.,* ***Phys. Earth Planet. Inter.****, 168, 237, 2008.*

Busse F.H. & Simitev R.D., ***Geophys. Astrophys. Fluid Dyn****., 100, 2006.*

- *Examples of linear oscillations*
- *Parker's plane layer theory of dynamo wave*
- *Non-linear oscillations*
- *Mechanism of excursions and reversals*

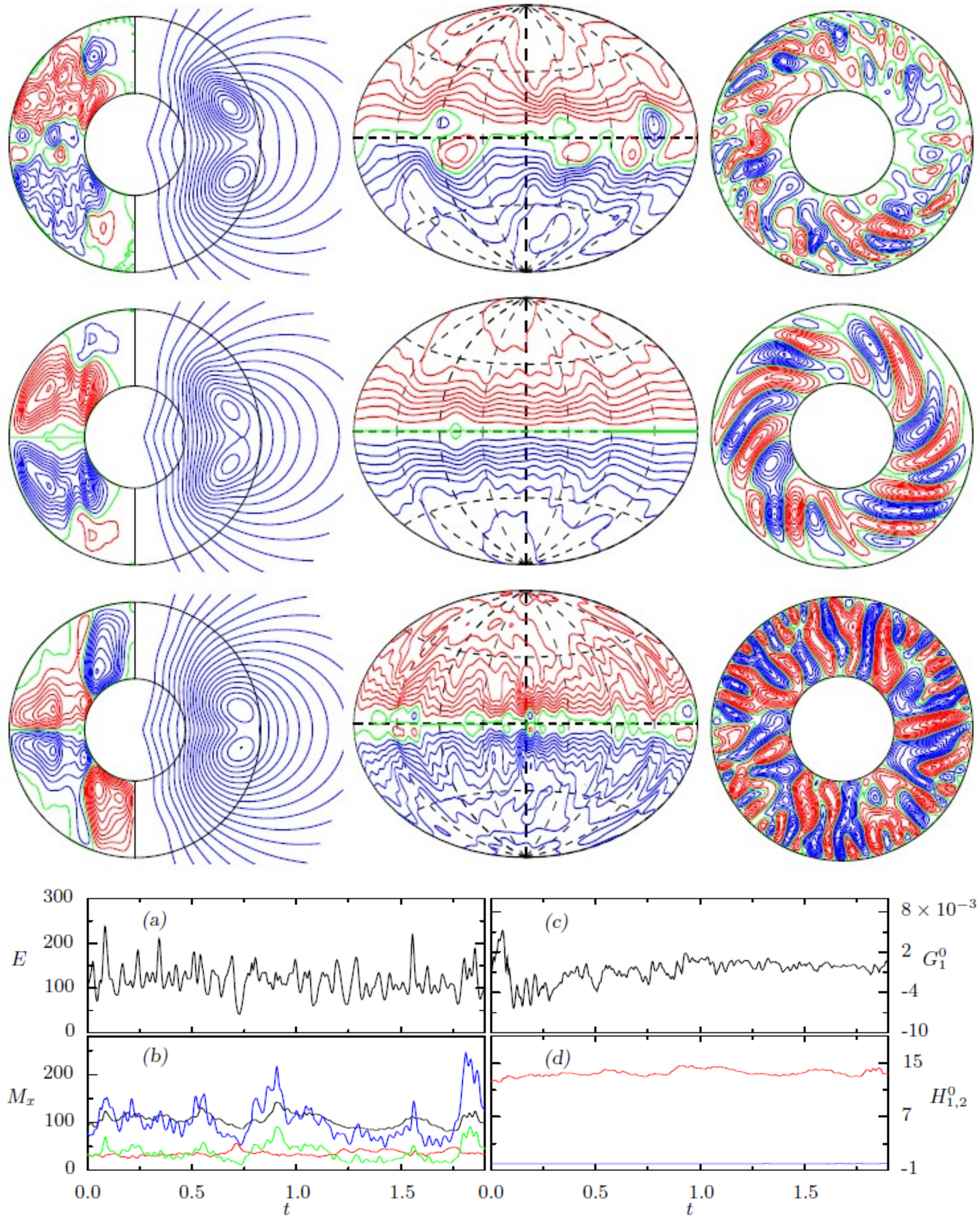
Non-oscillatory dynamos

A

B

C

B



Non-oscillatory dynamos:

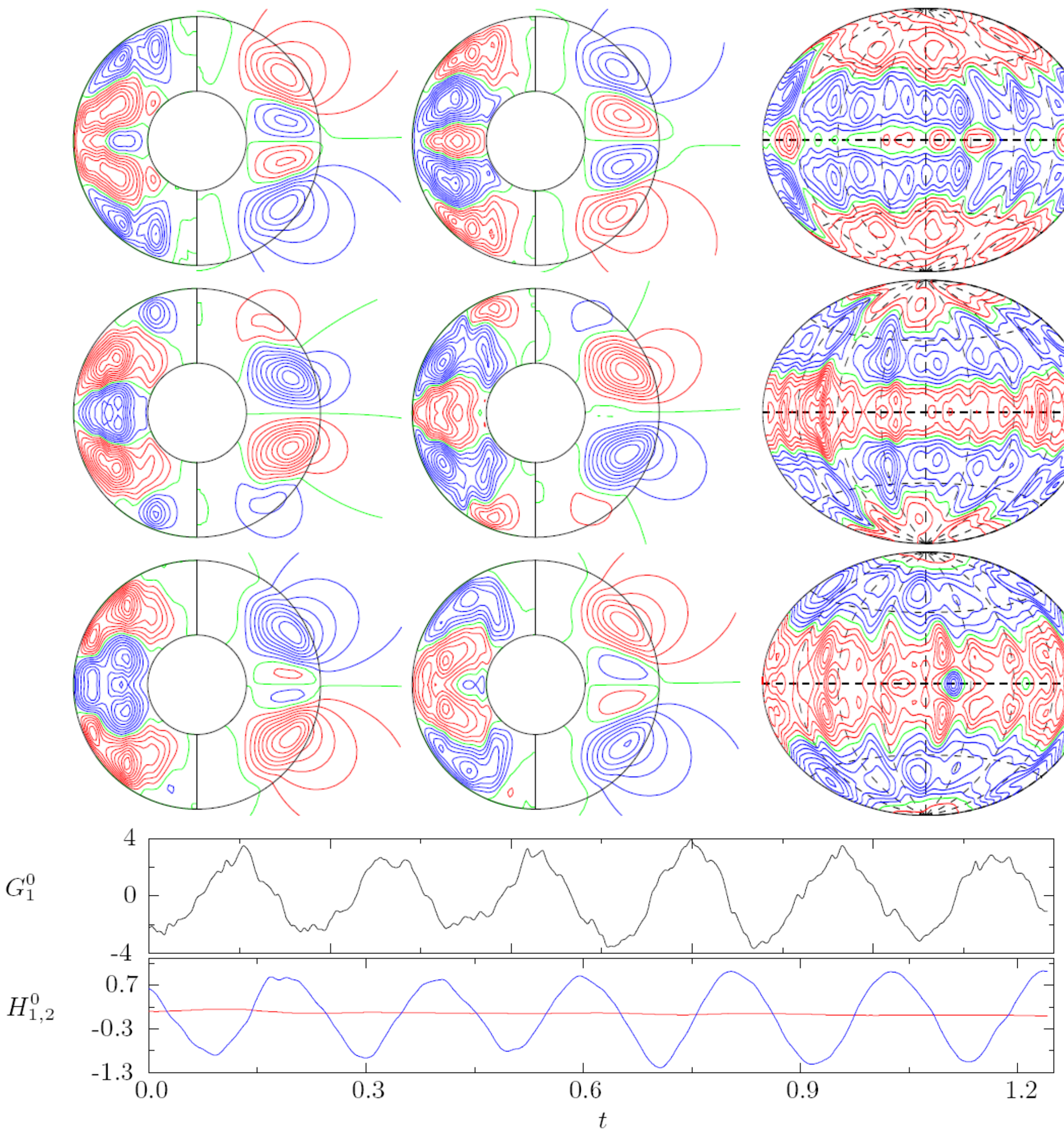
- *exist if the dipolar component is strongly dominant,*
- *have large ratio of Pm/P, so that quadrupolar components are not strong,*
- *are not too turbulent for otherwise higher harmonics will enter*

A $P = 0.1, \tau = 10^5, R = 3\times10^6,$
$Pm = 2$

B $P = 1, \tau = 10^4, R = 3.5\times10^5,$
$Pm = 10$

C $P = 200, \ \tau = 5\times10^3, \ R = 10^6,$
$Pm = 80$

Example of a quadrupolar oscillation



$$P = 5,\ \tau = 5 \times 10^3$$
$$R = 8 \cdot 10^5,\ P_m = 3$$

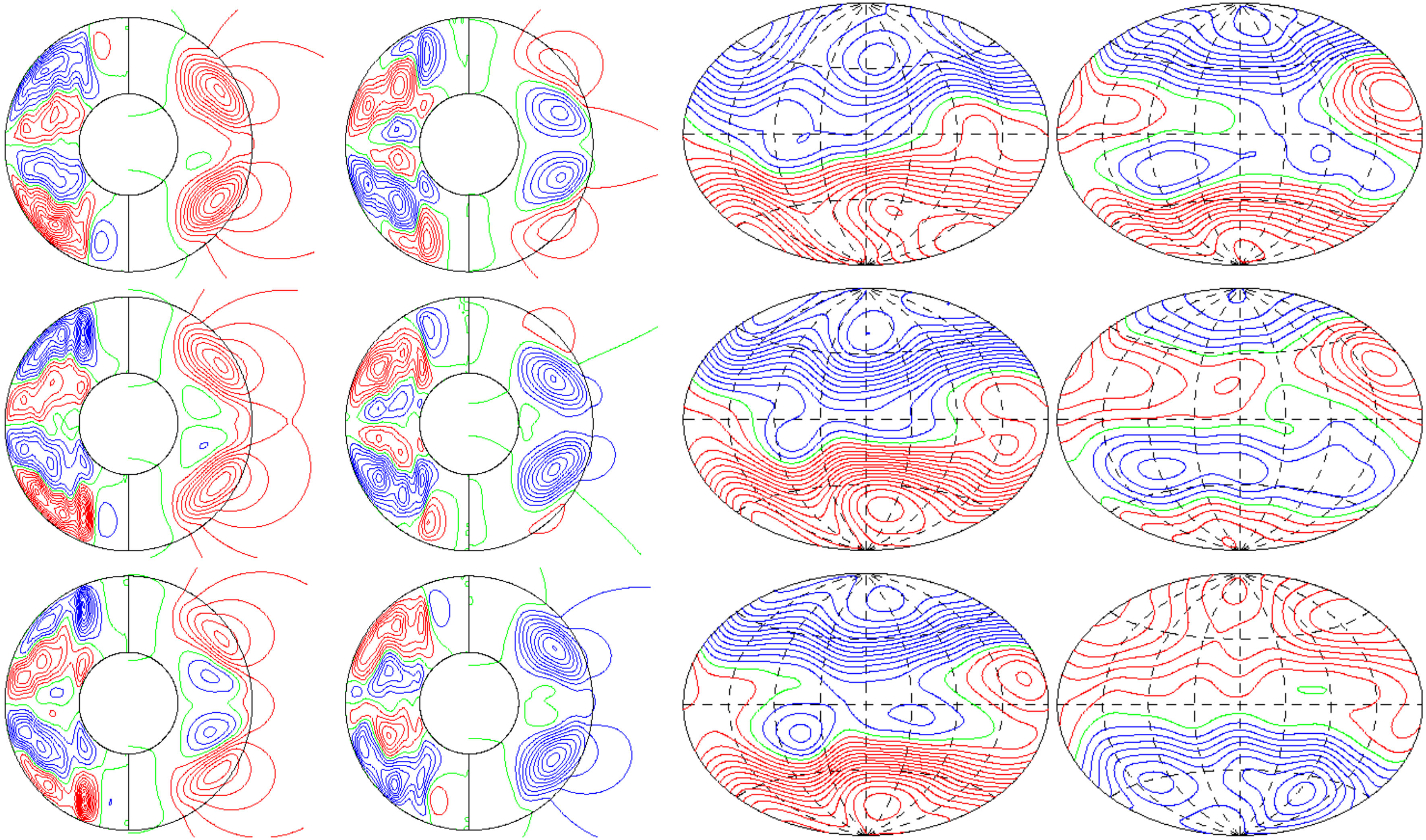
One period

Mean meridional filedlines of constant $\overline{B_\varphi}$ *(left),* $r \sin\vartheta \partial_\theta \overline{h}$ *(right) and radial magn. field.*

Time series of toroidal G_1^0 *and poloidal* H_1^0, H_2^0 *magn. coefficients.*

An example of a dipolar oscillation

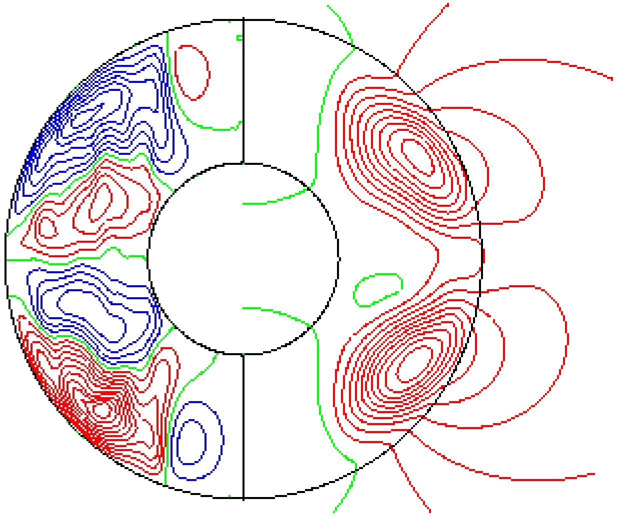
$$R = 3.5 \cdot 10^6,\ \tau = 3 \cdot 10^4,\ P = 0.75 \text{ and } P_m = 0.65$$

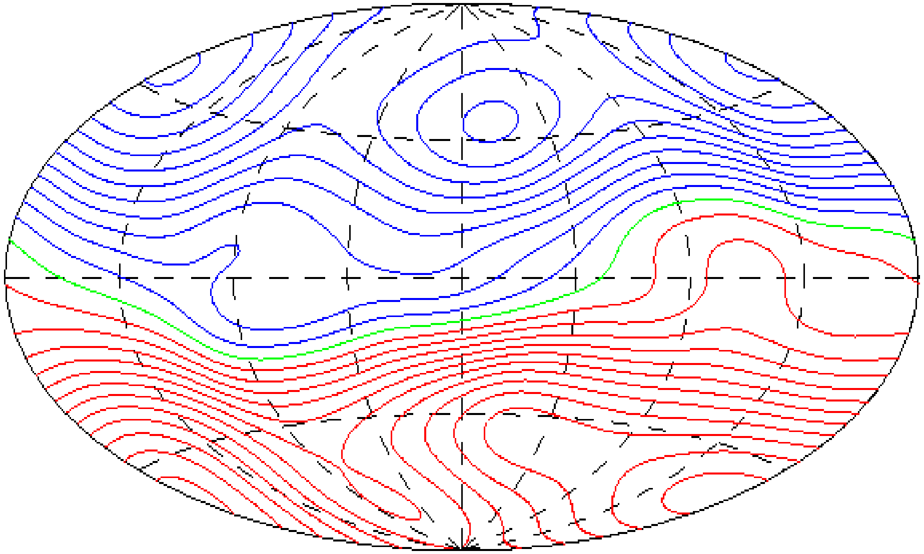


Half-period of oscillation (column-by-column)

An example of a dipolar oscillation

$$R = 3.5 \cdot 10^6,\ \tau = 3 \cdot 10^4,\ P = 0.75 \text{ and } P_m = 0.65$$





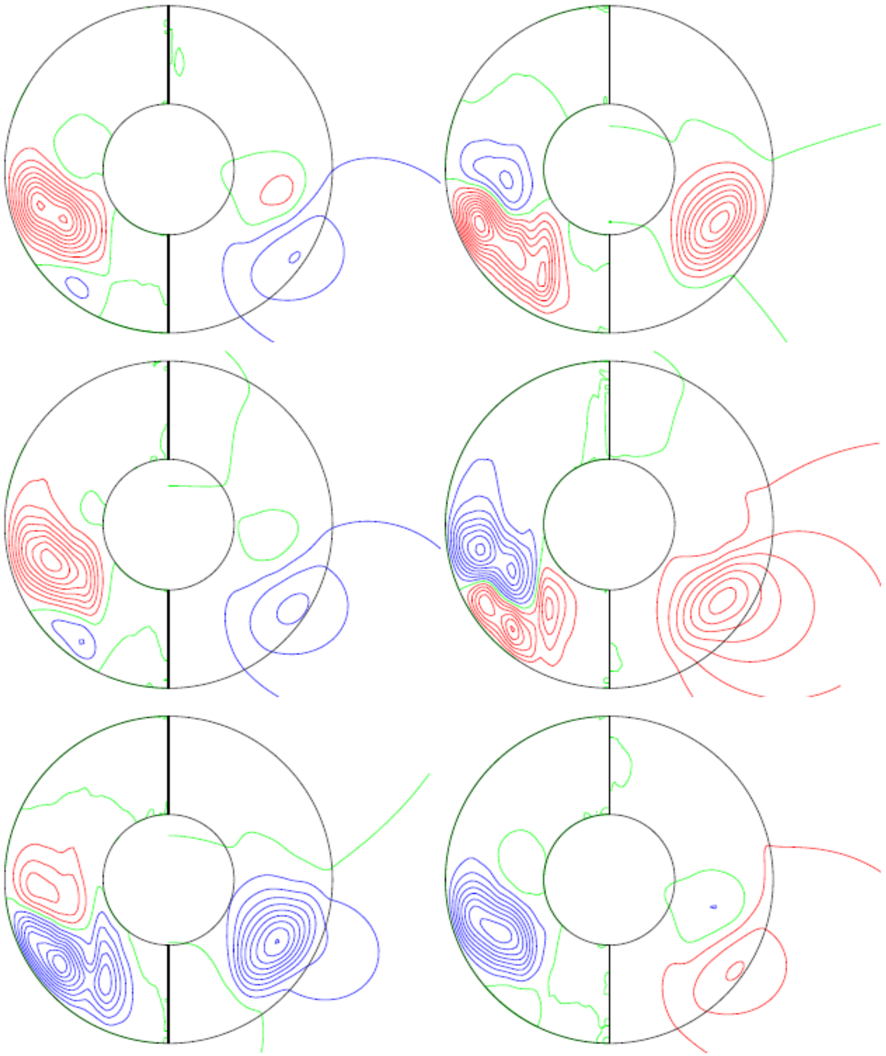
http://www.maths.gla.ac.uk/~rs/res/B/anim.bm.gif

http://www.maths.gla.ac.uk/~rs/res/B/anim.radmagn_2.gif

Half-period of oscillation (column-by-column)

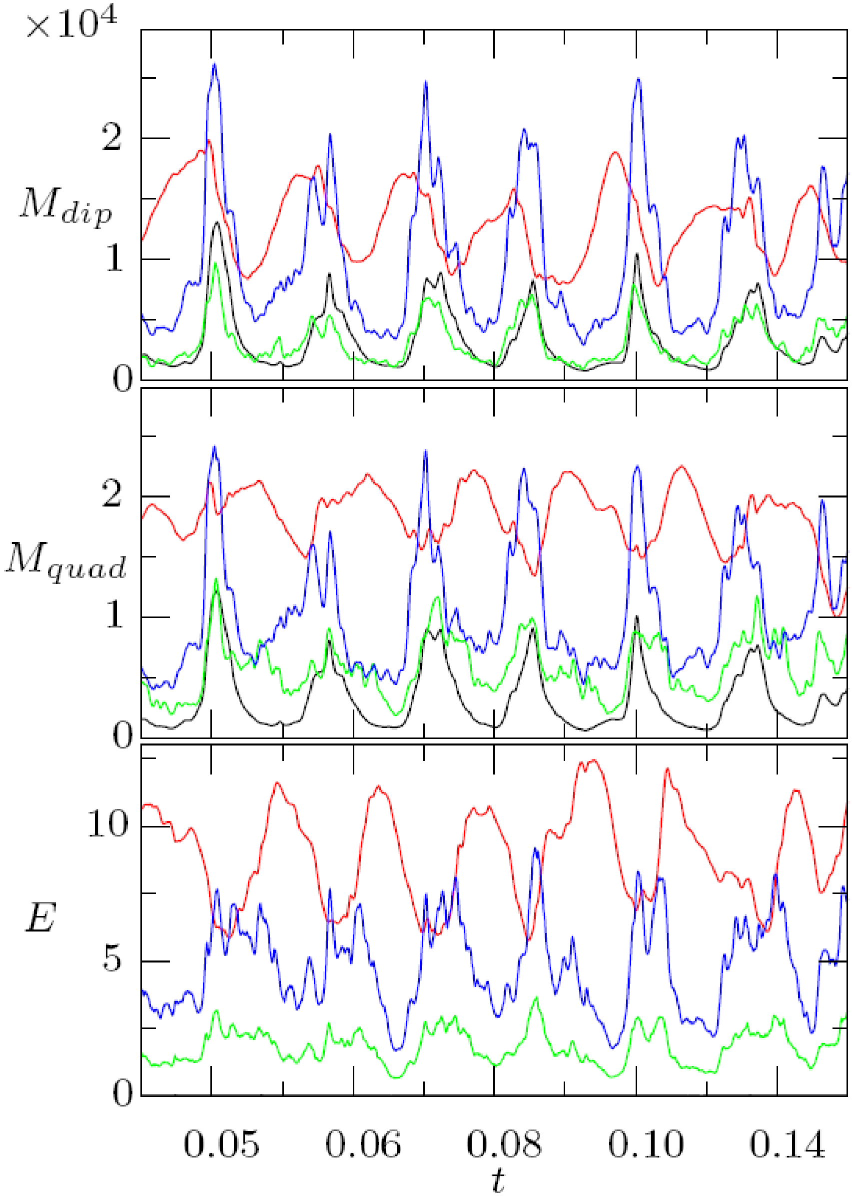
Effect of oscillations on convection

Mean meridional magn. fieldlines (clockwise)



$P = 0.1, \ \tau = 10^5, \ R = 6 \times 10^6, \ P_m = 0.11$

Energy densities



Linear Oscillations: Parker dynamo waves

Axisymmetric field:

$$\boldsymbol{B} = \boldsymbol{B}_p + \boldsymbol{i}B, \quad \boldsymbol{B}_p = \nabla \times \boldsymbol{i}A, \quad \boldsymbol{v} = \boldsymbol{i}\,U + \check{\boldsymbol{v}},$$

Following Parker's (1955) plane layer analysis of dynamo waves:

$$\frac{\partial}{\partial t}A = \hat{\alpha}B + \nabla^2 A/P_m, \quad \frac{\partial}{\partial t}B = \boldsymbol{B}_p \cdot \nabla U + \nabla^2 B/P_m,$$

Using a linear wave solution ansatz:

$$(A, B) = (\hat{A}, \hat{B}) \, \exp[i\boldsymbol{q} \cdot \boldsymbol{x} + \sigma t]$$

we can obtain an expression for the growth rate

$$\sigma = \begin{cases} -|\boldsymbol{q}|^2/P_m \pm \sqrt{\Gamma} \pm i\sqrt{\Gamma} & \text{for} \quad \Gamma > 0, \\ -|\boldsymbol{q}|^2/P_m \pm \sqrt{|\Gamma|} \mp i\sqrt{|\Gamma|} & \text{for} \quad \Gamma < 0 \end{cases}$$

$$\Gamma \equiv (-\hat{\alpha}(\boldsymbol{q} \times \nabla U)_x/2$$

$$\hat{q} \approx 2\pi/d$$

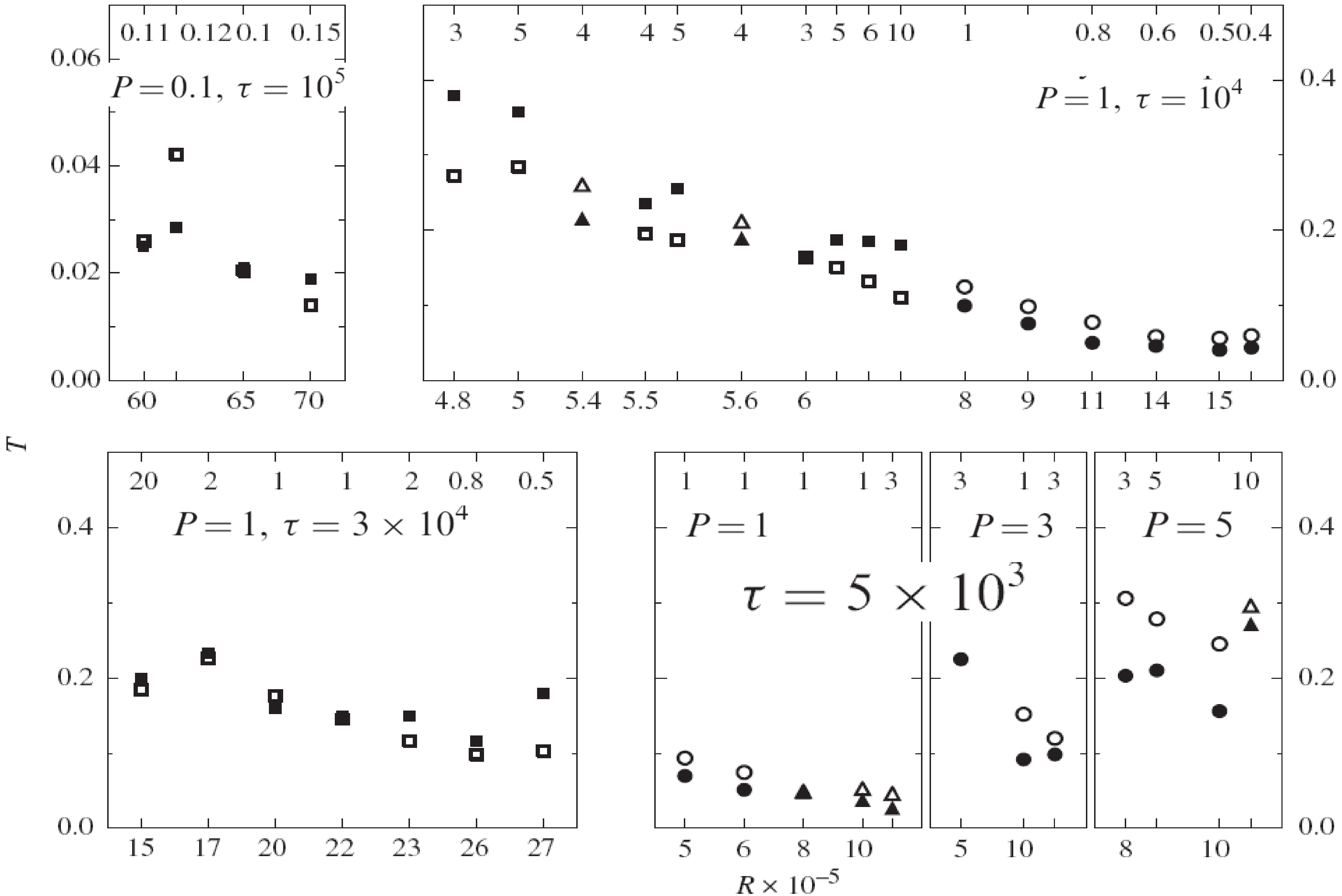
Assuming pseudo isotropic turbulence the alpha-coefficient is related to the helicity

$$\hat{\alpha} \equiv -\frac{1}{3P_m}\int\int \frac{\hat{q}^2 F(\hat{q},\omega)}{\omega^2 + \hat{q}^4/P_m^2} d\hat{q}d\omega \approx -\frac{P_m}{3\hat{q}^2}\int\int F(\hat{q},\omega)d\hat{q}d\omega \equiv -\frac{P_m}{3\hat{q}^2}\langle \check{\boldsymbol{v}} \cdot \nabla \times \check{\boldsymbol{v}} \rangle$$

Period:

$$T \approx 4\pi^2 \left(P_m \frac{\pi}{3} \langle \check{\boldsymbol{v}} \cdot \nabla \times \check{\boldsymbol{v}} \rangle \sqrt{2\ \overline{E_t}} \right)^{-1/2}$$

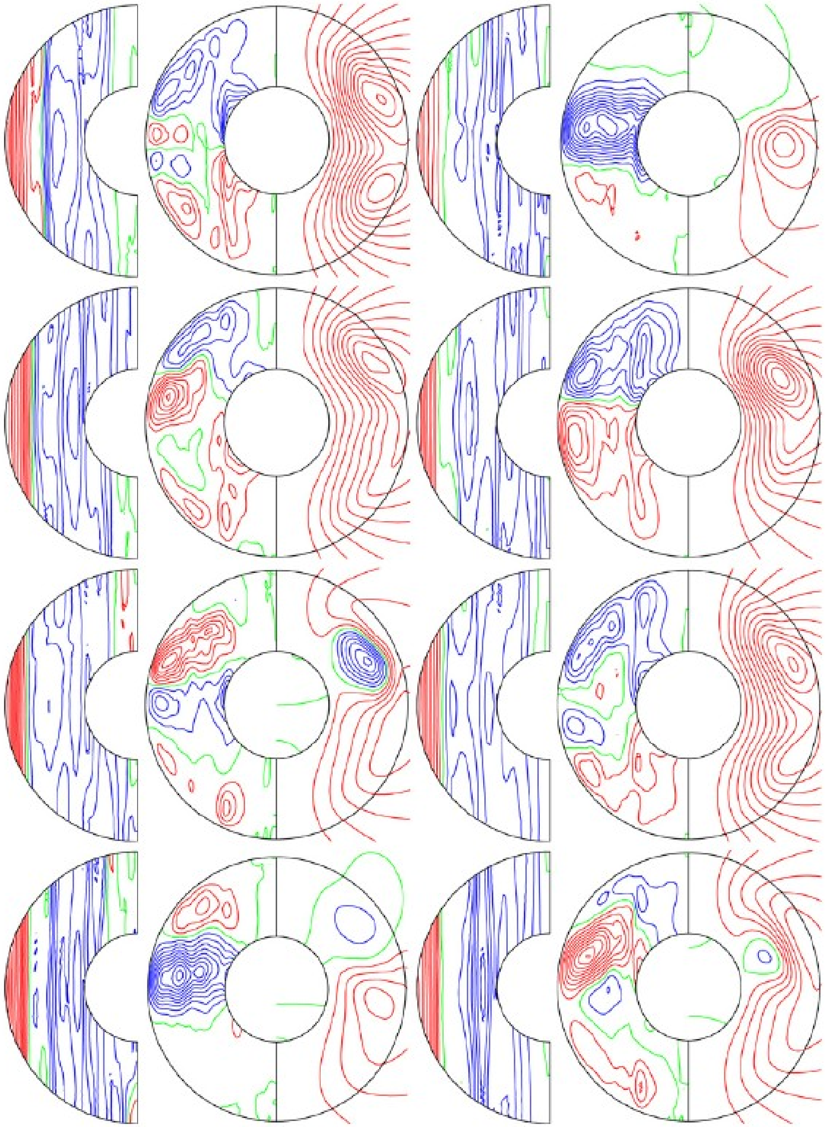
Period of oscillations: model vs. numerics



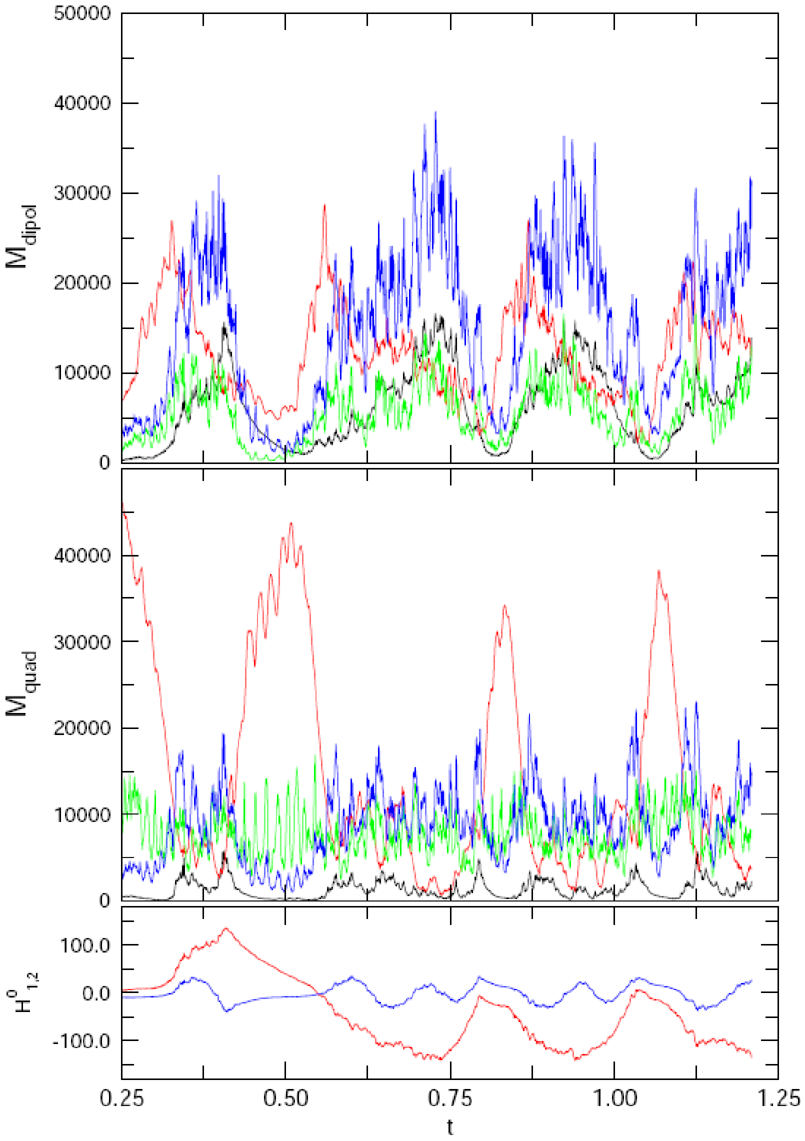
Non-linear dynamo oscillations

$$P = 0.1, \tau = 10^5, R = 4 \times 10^6, P_m = 0.5$$

Mean meridional magn. fieldlines (clockwise)

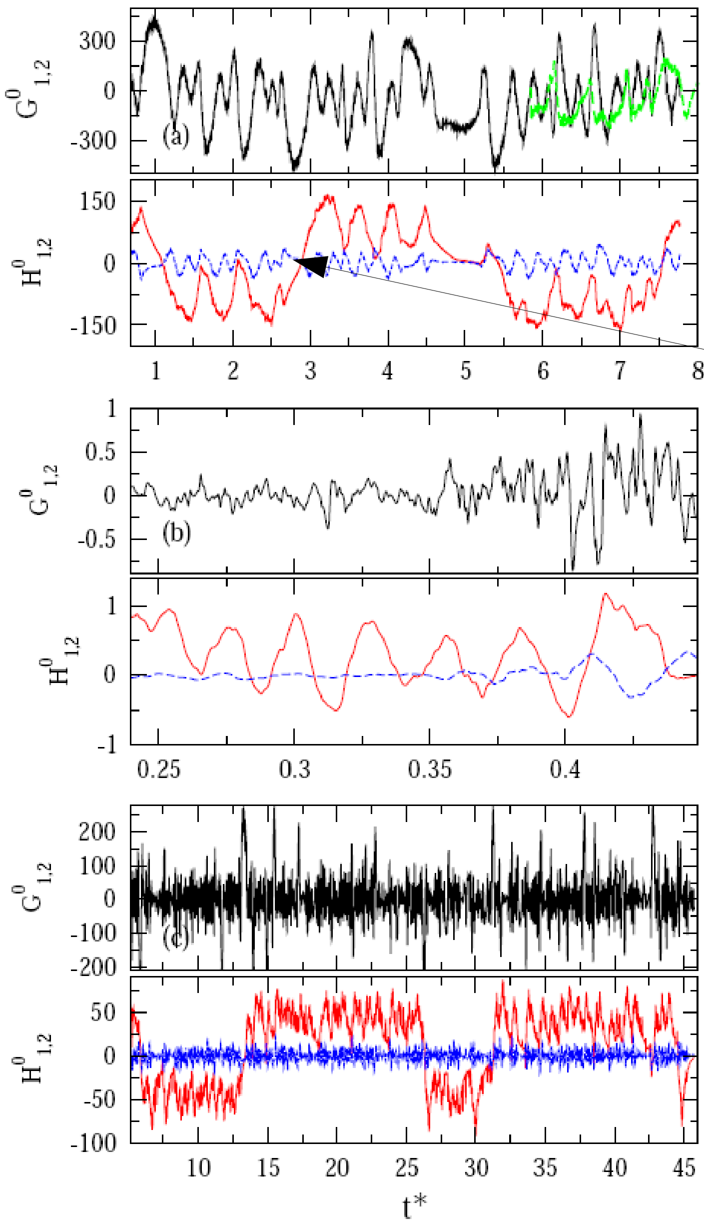


Energy densities



Busse & Simitev, PEPI, 2008

Reversals cased by toroidal flux oscillations

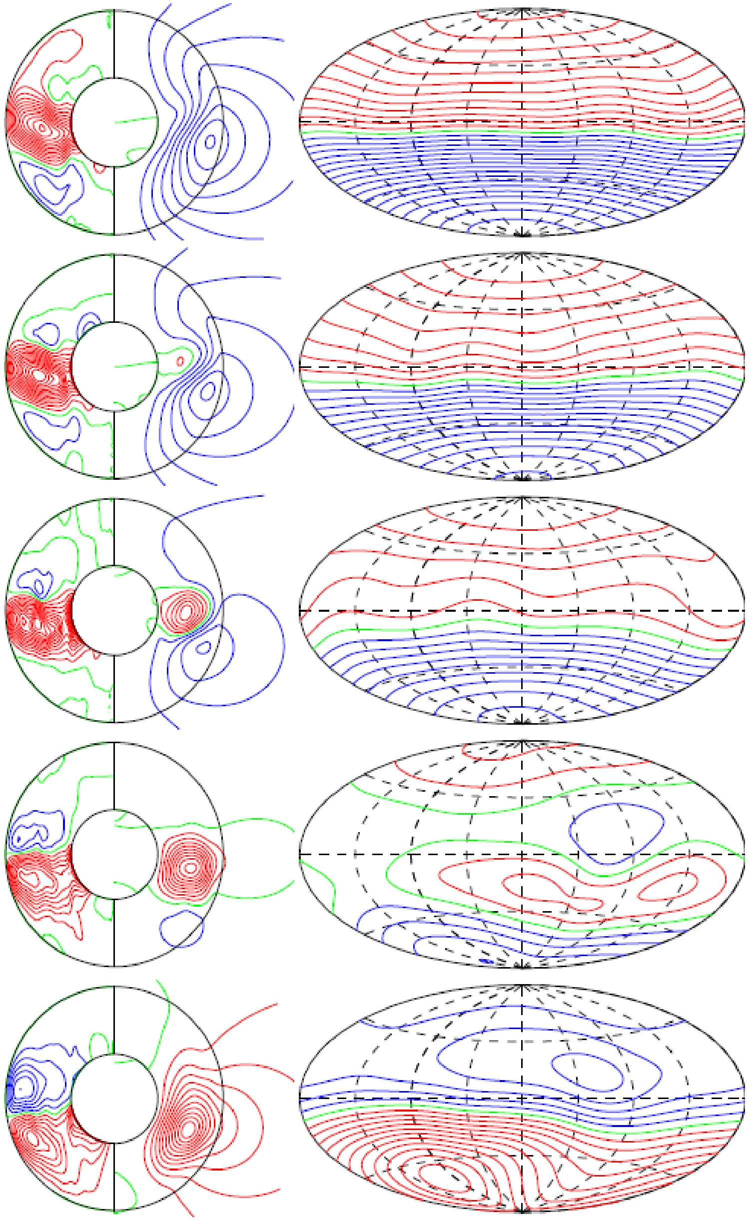


$$P = 0.1,\ \tau = 10^5$$
$$R = 4 \times 10^6,\ P_m = 0.5$$

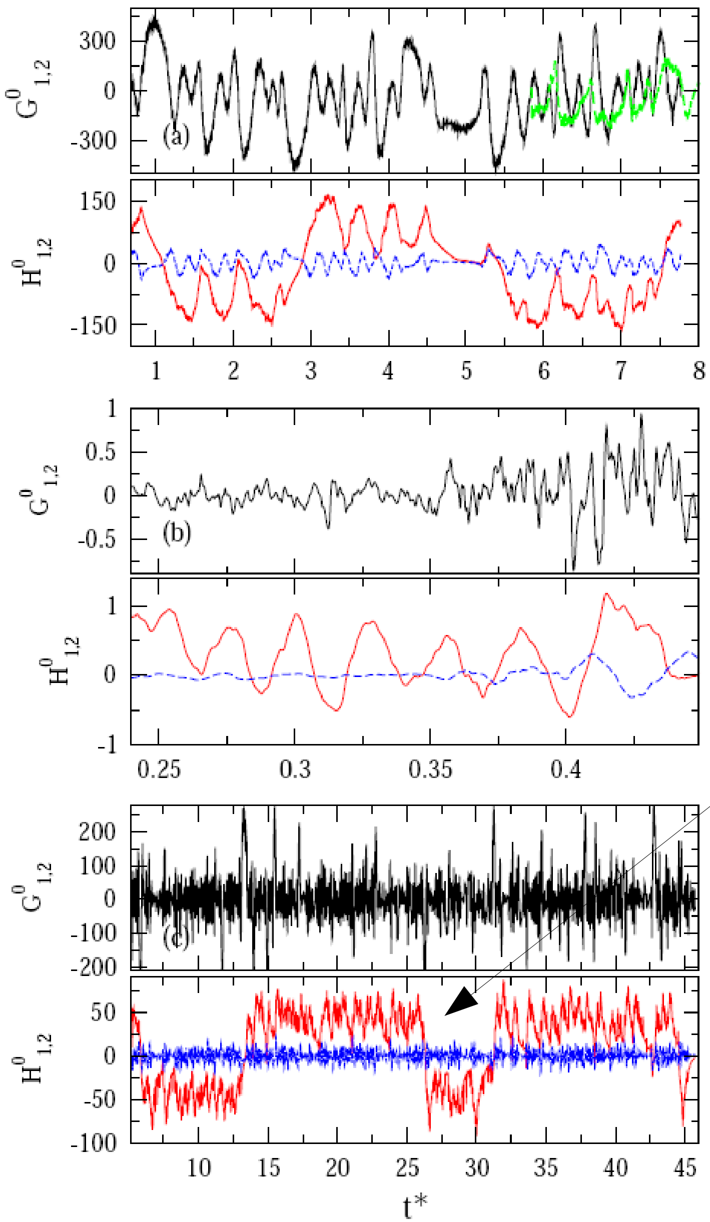
Oscillating FD dynamo

$$P = 0.1,\ \tau = 3 \times 10^4$$
$$R = 850000 \quad P_m = 1$$

Busse & Simitev, PEPI, 2008



Reversals cased by toroidal flux oscillations



$$P = 0.1,\ \tau = 10^5$$
$$R = 4 \times 10^6,\ P_m = 0.5$$

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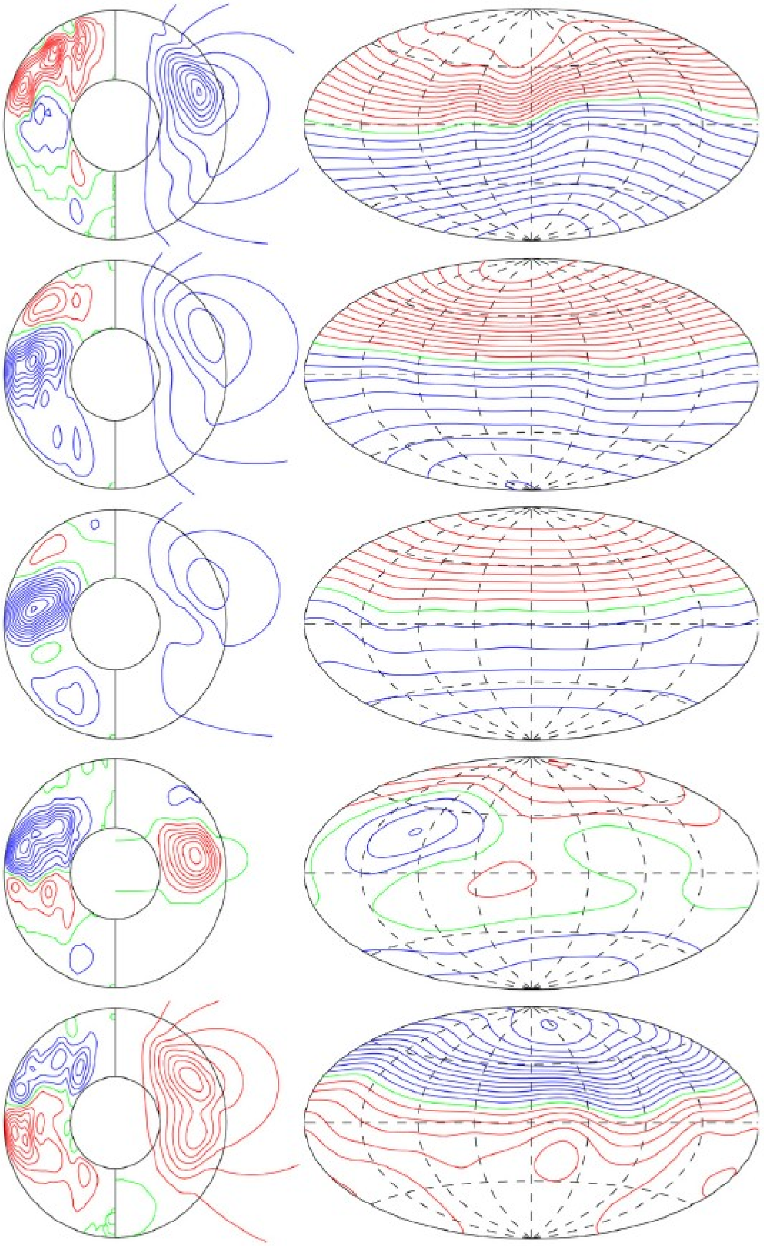
Busse & Simitev, PEPI, 2008



Some similarities with geomagnetic observations

$$P = 0.1,\ \tau = 10^5$$
$$R = 4 \times 10^6,\ P_m = 0.5$$

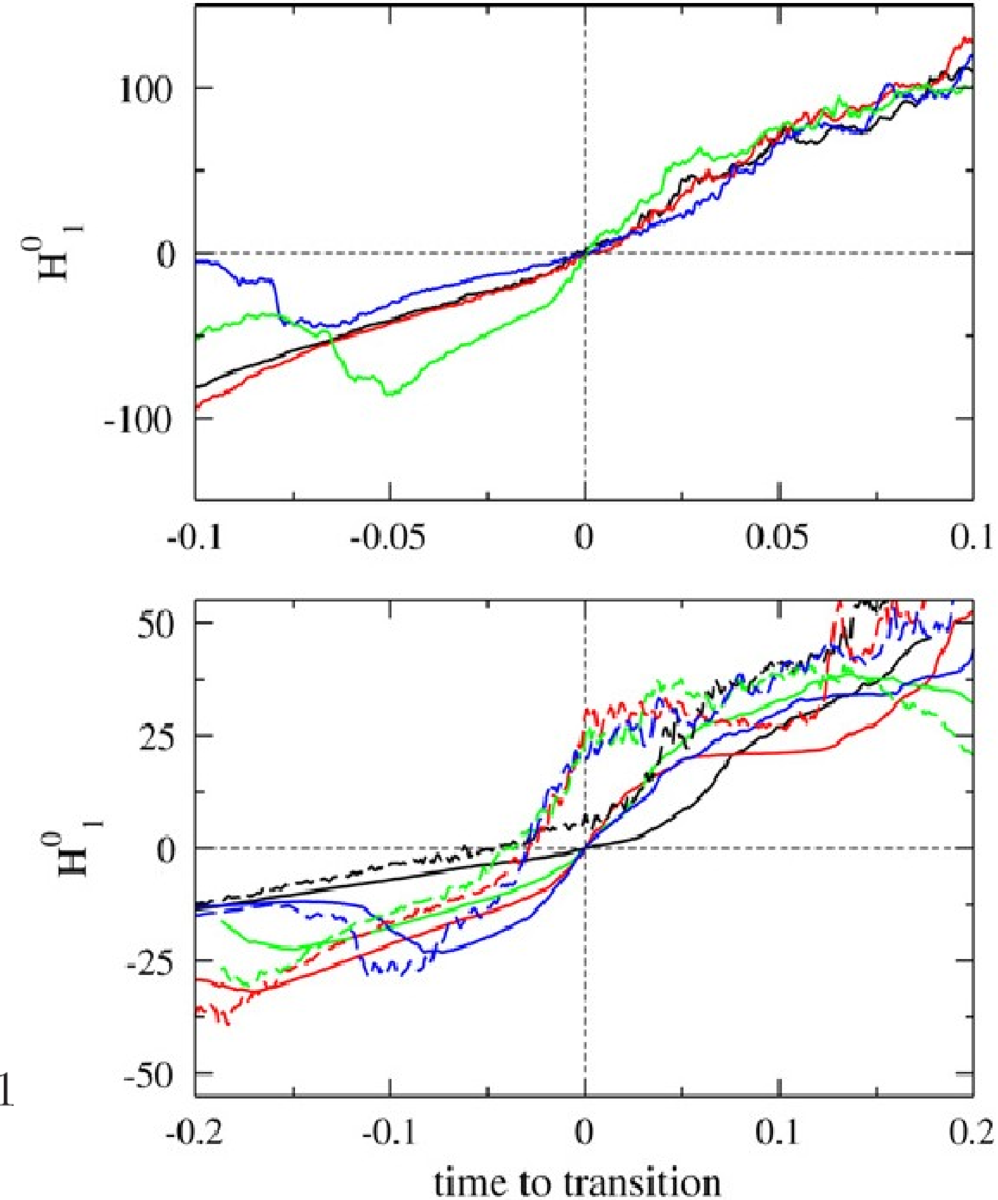
1) Reversed magnetic field appears first at low latitudes as also observed by Clement (Nature, 2004).

2) Average duration of a reversal event is ~20000yrs - roughly consistent with observations.

3) For each reversal we observe several excursion events.

4) Amplitude of the field increases more sharply after a reversal than than it decays before the reversal.

$$P = 0.1,\ \tau = 3 \times 10^4$$
$$R = 850000 \quad P_m = 1$$



Busse & Simitev, PEPI, 2008

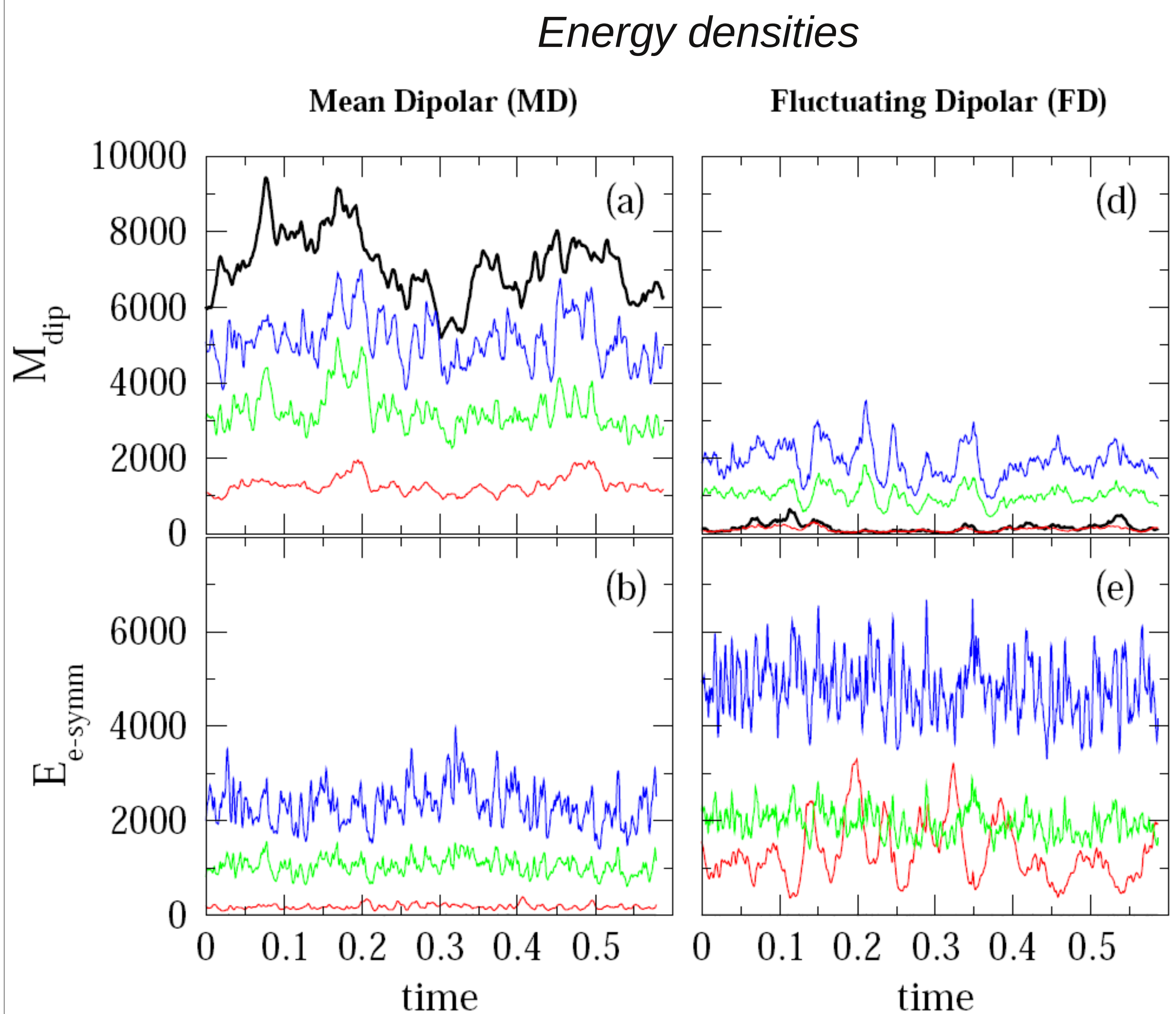
Part IV

Bistability and hysteresis of dipolar dynamos generated by chaotic convection in rotating spherical shells

Ref: *Simitev, R., Busse F.H.,* ***Europhysics Letters****, 85, 19001, 2009.*

- *Two types of dipolar dynamos generated by chaotic convection at identical external parameter values*
- *The transition between Mean Dipolar (MD) and Fluctuating Dipolar (FD) dynamos and the hysteresis phenomenon*
- *Contrasting properties of Mean Dipolar (MD) and Fluctuating Dipolar (FD) dynamos*
- *Oscillations of Fluctuating Dipolar (FD) dynamos and reversals*

Two types of dipolar dynamos generated by chaotic convection



• ***Fully chaotic*** (***large-scale turbulent***) *regime.*

• ***Two chaotic attractors*** *for the same parameter values.*

• *Essential qualitative difference: contribution of the* ***mean poloidal dipolar energy***

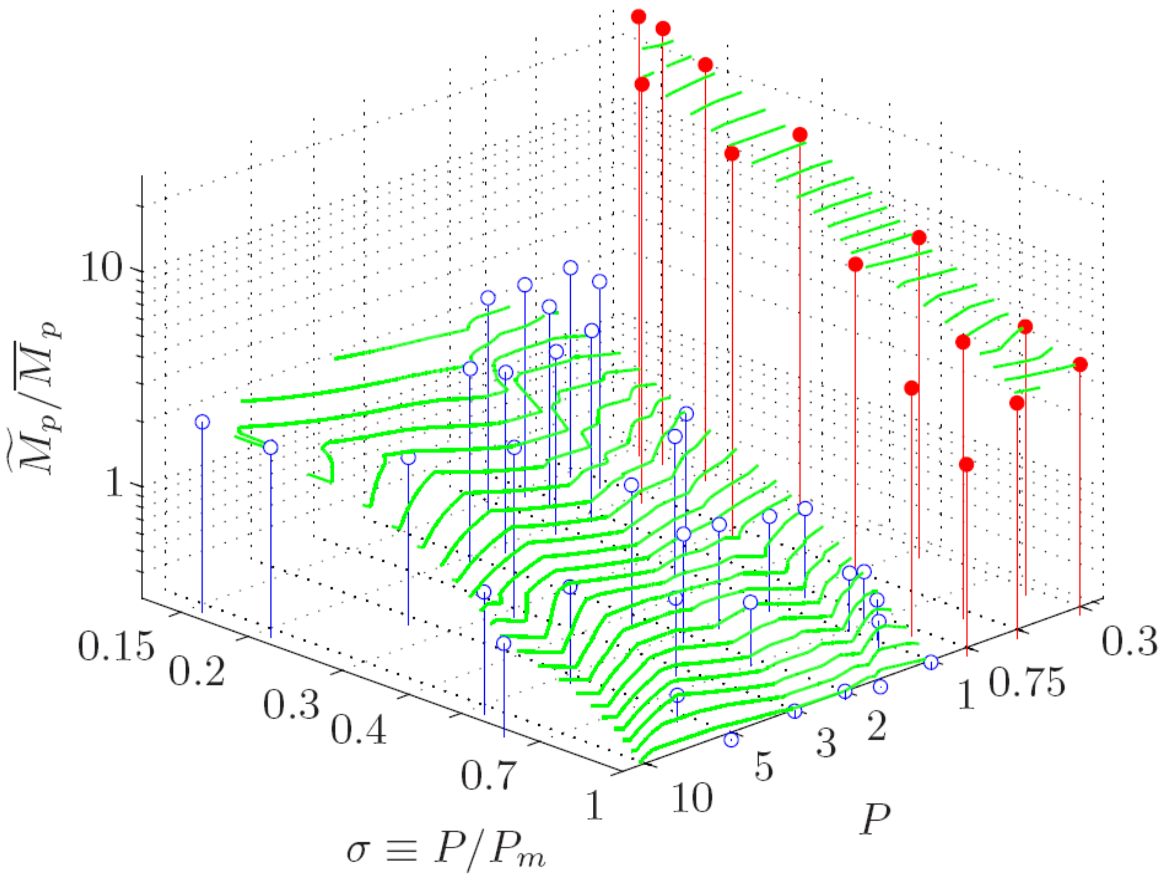
	(ab)	(de)
Rm	133.6	196.5
Mdip/Mtot	0.803	0.527

*black.......**mean poloidal***
*green......**fluctuating poloidal***
*red..........**mean toroidal***
*blue.........**fluctuating toroidal***

$R = 3.5 \cdot 10^6$, $\tau = 3 \cdot 10^4$, $P = 0.75$ and $P_m = 1.5$

Regions and transition

Ratio of ***fluctuating to mean poloidal magn energy***



$$R = 3.5 \cdot 10^6, \ \tau = 3 \cdot 10^4$$

Two types of dipolar dynamos

Mean Dipolar (MD)

$$\widetilde{M}_p < \overline{M}_p$$

Fluctuating Dipolar (FD)

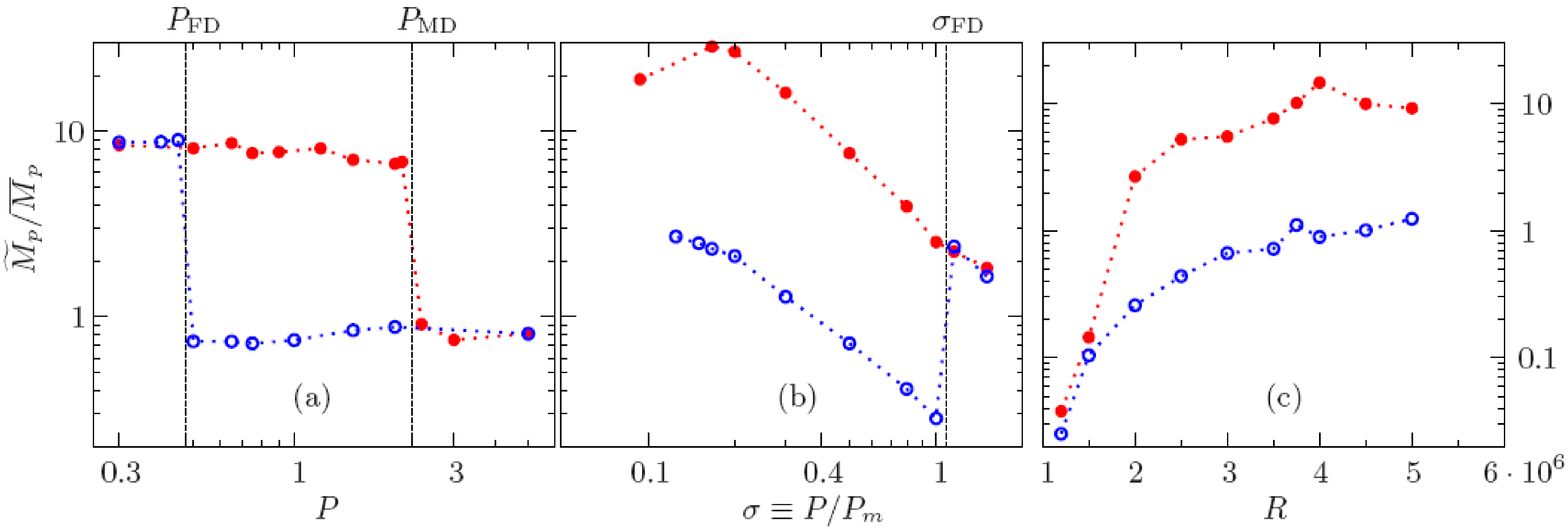
$$\widetilde{M}_p > \overline{M}_p$$

- *MD and FD dynamos correspond to rather* ***different chaotic attractors*** *in a fully chaotioc system*
- *The transition between them is not gradual but is an* ***abrupt jump*** *as a critical parameter value is surpassed.*
- *The nature of the transition is complicated.*

	MD	FD
Mdip/Mtot	(0.62,1)	(0.41,56)

Bistability and hysteresis in the MD <==> FD transition

***Bistability and hysteresis** in the ratio of fluctuating poloidal to mean poloidal magn energy*



(a) $R = 3.5 \cdot 10^6 \quad P/P_m = 0.5$

(b) $R = 3.5 \cdot 10^6, \; P = 0.75$

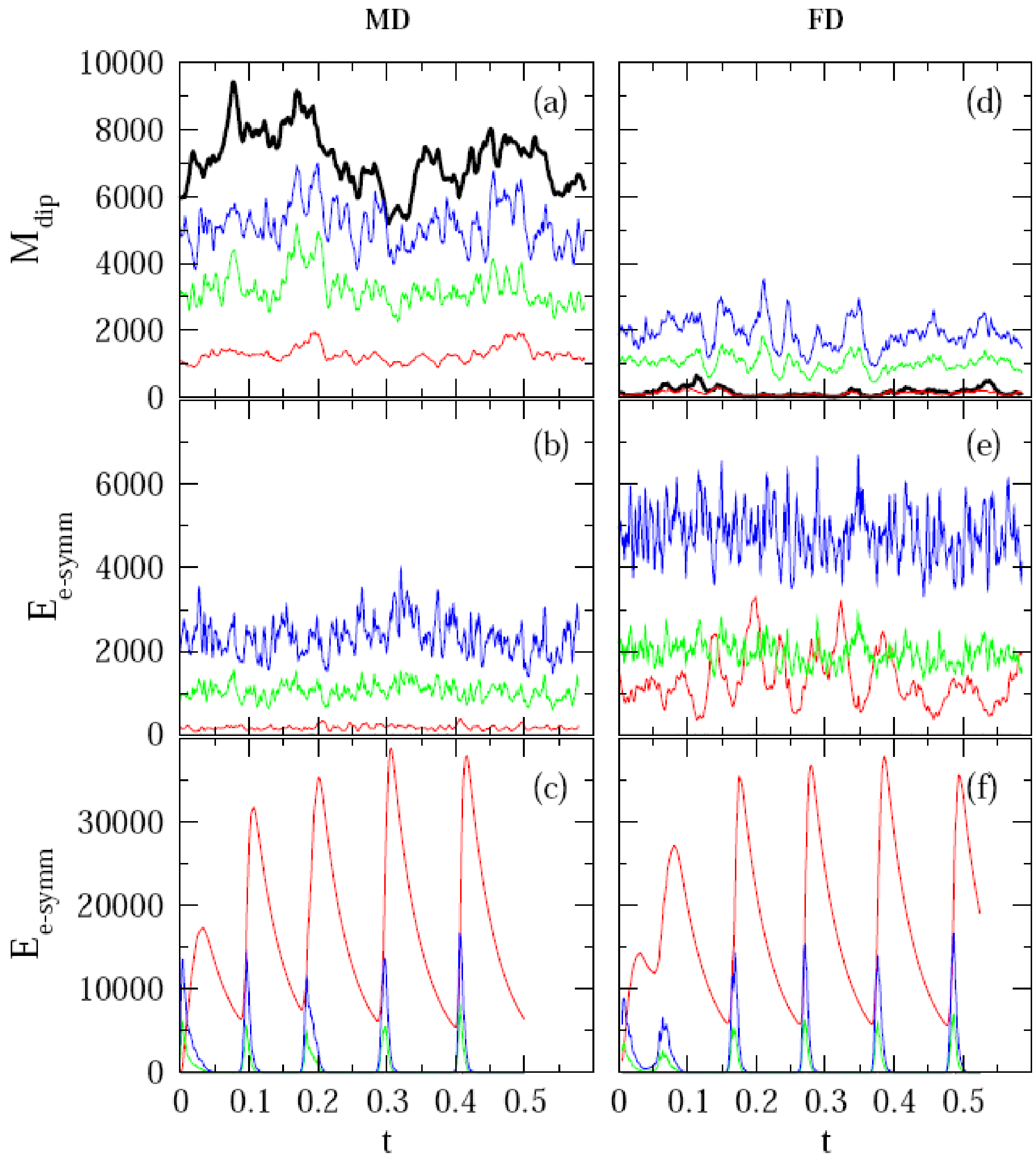
(c) $P = 0.75, \; P_m = 1.5$

in all cases: $\tau = 3 \cdot 10^4$

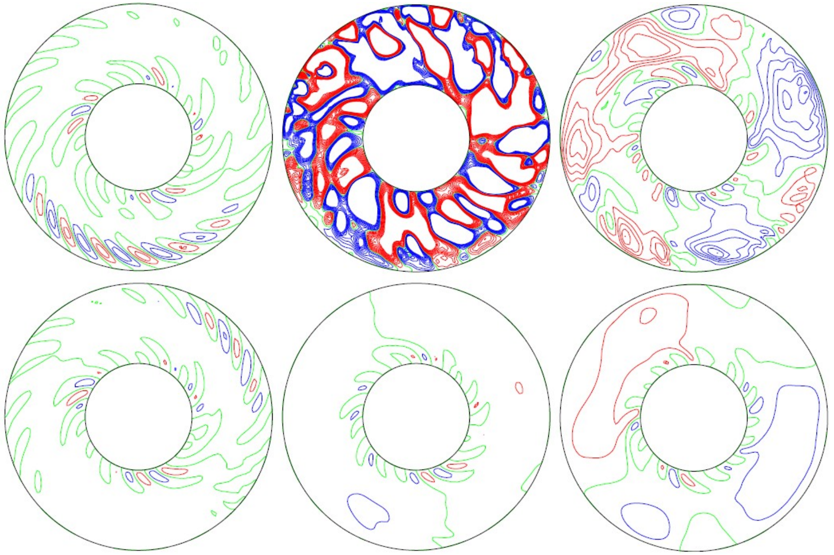
The coexistence is
not an isolated phenomenon
but can be traced with
variation of the parameters.

$P_{MD} = 2.2$ $\quad P_{FD} = 0.5$
$\sigma_{MD} = 0.07$ $\quad \sigma_{FD} = 1$

The hysteresis is a purely magnetic effect



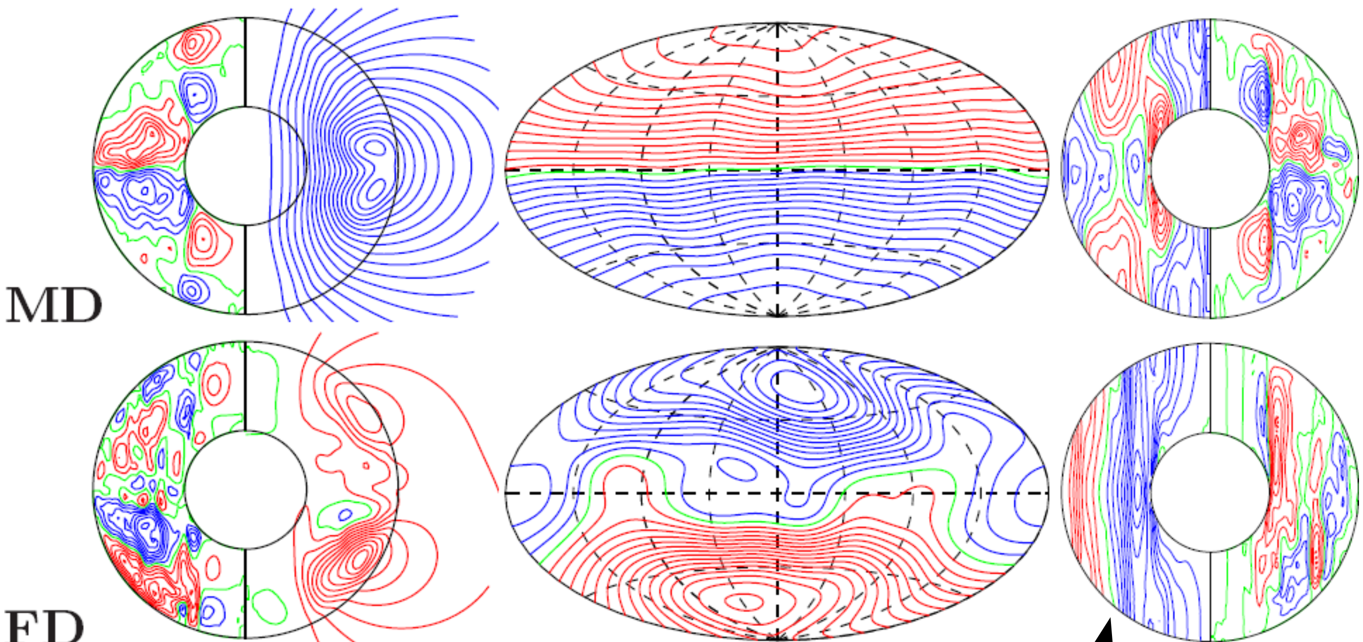
After magnetic field is suppressed both MD and FD dynamos equilibrate to statistically ***identical convective states*** *(period of relaxation oscillations, clockwise)*



Magnetic field is artificially suppressed, i.e. ***non-magnetic convection***

$$R = 3.5 \cdot 10^6, \ \tau = 3 \cdot 10^4 \quad P = 0.75 \text{ and } P_m = 1.5$$

A property comparison of MD and FD dynamos (Spatial structures)



Snapshots of spatial structures

$$R = 3.5 \times 10^6,\ \tau = 3 \times 10^4$$
$$P = 0.75 \text{ and } P_m = 1.5$$

The stronger magnetic field of MD dynamos counteracts differential rotation (diff rotation, meridional streamlines)

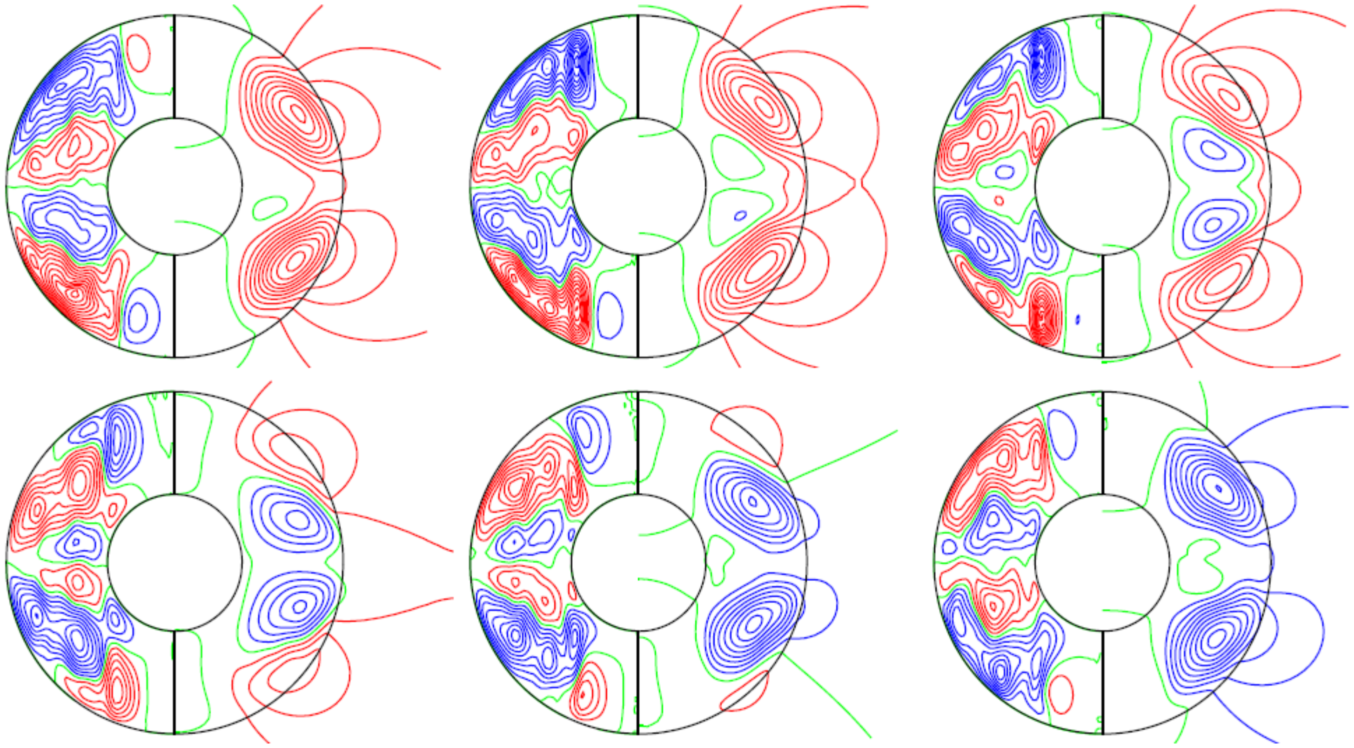
Both MD and FD dynamos appear dipolar from the outside (radial magn field at r=ro+1.3, Earth's surface)

FD dynamos have a somewhat more irregular and small-scale internal structure (Bphi and meridional fieldlines)

A property comparison of MD and FD dynamos (Temporal variations)

- ***Mean Dipolar*** *(MD) dynamos are* ***non-oscillatory.***
- ***Fluctuating Dipolar*** *(FD) dynamos are* ***oscillatory.***

Half-period of oscillation in a FD dynamo (row-by-row)



$$R = 3.5 \cdot 10^6,\ \tau = 3 \cdot 10^4,\ P = 0.75 \text{ and } P_m = 0.65$$

Conclusion

- *We have described typical properties of self-consistent spherical dynamos in rotating spherical shells.*

- *Dynamo oscillations are typical temporal behaviour,*

- *Oscillations may lead to aperiodic reversals similar to geomagnetic reversals.*

- *Co-existence of chaotic attractors.*