# Magnetism and Thermal Evolution of the Terrestrial Planets<sup>1</sup>

# DAVID J. STEVENSON,\* TILMAN SPOHN,<sup>+,2</sup> AND GERALD SCHUBERT<sup>†</sup>

\*Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, California 91125, and †Department of Earth and Space Sciences, University of California, Los Angeles, Los Angeles, California 90024

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Of the terrestrial planets, Earth and probably Mercury possess substantial intrinsic magnetic fields generated by core dynamos, while Venus and Mars apparently lack such fields. Thermal histories are calculated for these planets and are found to admit several possible present states, including those which suggest simple explanations for the observations; while the cores of Earth and Mercury are continuing to freeze, the cores of Venus and Mars may still be completely liquid. The models assume whole mantle convection, which is parameterized by a simple Nusselt-Rayleigh number relation and dictates the rate at which heat escapes from the core. It is found that completely fluid cores, devoid of intrinsic heat sources, are not likely to sustain thermal convection for the age of the solar system but cool to a subadiabatic, conductive state that can not maintain a dynamo. Planets which nucleate an inner core continue to sustain a dynamo because of the gravitational energy release and chemically driven convection that accompany inner core growth. The absence of a significant inner core can arise in Venus because of its slightly higher temperature and lower central pressure relative to Earth, while a Martian core avoids the onset of freezing if the abundance of sulfur in the core is  $\gtrsim 15\%$  by mass. All of the models presented assume that (i) core dynamos are driven by thermal and/or chemical convection; (ii) radiogenic heat production is confined to the mantle; (iii) mantle and core cool from initially hot states which are at the solidus and superliquidus, respectively; and (iv) any inner core excludes the light alloying material (sulfur or oxygen) which then mixes uniformly upward through the outer core. The models include realistic pressure and composition-dependent freezing curves for the core, and material parameters are chosen so that the correct present-day values of heat outflow, upper mantle temperature and viscosity, and inner core radius are obtained for the Earth. It is found that Venus and Mars may have once had dynamos maintained by thermal convection alone. Earth may have had a completely fluid core and a dynamo maintained by thermal convection for the first 2 to 3 by, but an inner core nucleates and the dynamo energetics are subsequently dominated by gravitational energy release. Complete freezing of the Mercurian core is prohibited if it contains even a small amount of sulfur, and a dynamo can be maintained by chemical convection in a thin, fluid shell.

#### I. INTRODUCTION

The terrestrial planets Mercury, Venus, Earth, and Mars are strikingly different in their magnetism. Earth's surface magnetic field is dominated by a strong dipole component (a moment of  $8 \times 10^{22}$  A m<sup>2</sup>). Conventional wisdom has it that the field is produced by regenerative dynamo action in the Earth's iron-rich fluid outer core. Fluid motions of the highly conducting liquid in the presence of the magnetic field induce currents which themselves generate the field. The fluid motions may be due to a variety of causes, including thermally or chemically driven convection. Chemically driven convection can arise from the release of buoyant light material upon freezeout of a solid inner core from an outer core of noneutectic composition (Braginsky, 1964). Latent heat release also occurs with inner core growth (Verhoogen, 1961). A dynamo driven by chemical buoyancy may have a much higher thermodynamic efficiency

<sup>&</sup>lt;sup>1</sup> Contribution 3820 from the Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, Calif. 91125.

<sup>&</sup>lt;sup>2</sup> Permanent address: Institut für Meteorologie und Geophysik, J. W. Goethe Universität, D-6000, Frankfurt, Feldbergstrasse 47, West Germany.

than one driven thermally because the Carnot efficiency factor is not involved (Gubbins, 1977a; Loper, 1978). It is also possible, although less likely, that motions driven by precession can contribute to dynamo generation (Malkus, 1963; Stevenson, 1974; Rochester *et al.*, 1975; Loper, 1975; Busse, 1978). For a general review of the dynamo problem, see Gubbins (1974), Levy (1976), Busse (1978), Moffatt (1978), and Parker (1979). For a review of planetary magnetic field observations, see Ness (1979) and Russell (1980); for a synthesis of their relationship to the dynamo problem, see Stevenson (1983).

Despite its similarity in size to Earth, Venus does not possess a significant dipole magnetic field. Early data from U.S. and USSR spacecraft (Venera 4, Dolginov et al., 1969; Mariner 5, Bridge et al., 1969) were interpreted by Russell (1976a,b) to give an upper bound to a possible Venusian magnetic dipole moment of  $6.5 \times 10^{19}$  A m<sup>2</sup>. Data from Venera 9 and 10 lowered the upper bound estimate to 4  $\times$  10<sup>19</sup> A m<sup>2</sup> (Dolginov et al., 1978). Pioneer Venus orbiter data have further reduced this estimate to  $5 \times 10^{18}$  A m<sup>2</sup> (Russell *et al.*, 1980). Venus has a similar mass and radius to Earth and is commonly assumed to have a metallic core. This assumption is unconstrained by gravity field and flattening observations because Venus rotates slowly and nonhydrostatic effects dominate. However, the rotation is fast enough that Coriolis effects are very important for largescale motions in the assumed core. This appears to have been noticed first by Hide (1956). Although a predictive dynamo theory does not yet exist, the importance of Coriolis effects in a liquid core suggests that if there exist sources of fluid motion comparable to those in the Earth's core. then Venus should have a dynamo. The very small observational upper bound to the magnetic dipole moment argues against a dynamo and implies that the core of Venus is different from the Earth in respects other than merely a very different rotation rate.

Interpretations of magnetic field data from Mars spacecraft are controversial. Dolginov (1977, 1978a,b) interprets the data from Mars 2, 3, and 5 in terms of an intrinsic magnetic field with dipole moment of about  $2.5 \times 10^{19}$  A m<sup>2</sup>. However, Russell (1978a,b) concludes that these data are consistent with a magnetosphere induced by the interaction of Mars with the solar wind. According to Ness (1979), the data are inadequate to allow more than a speculative interpretation. The Viking retarding potential analyzer data have been interpreted as suggesting a small permanent field (Intriligator and Smith, 1979; Cragin et al., 1982). In any event, the field is much smaller than one would expect from an active dynamo in a rapidly rotating planet (Stevenson, 1983). Estimates of Mars' moment of inertia suggest that Mars possesses a metallic core (Anderson, 1975; Reasenberg, 1977; Bills and Ferrari, 1978; Kaula, 1979). The inferred absence of a dynamo suggests that the Martian core lacks sources of motion similar to those available in Earth's core.

Unlike Mars and Venus, Mercury appears to have a substantial intrinsic magnetic field (Ness et al., 1974, 1975, 1976) with a dipole moment of between 2.8 and  $4.9 \times 10^{19}$  A m<sup>2</sup>, but with considerable uncertainty in alignment and higher order multipoles (Slavin and Holzer, 1979). The high density of Mercury suggests that it has a large iron-rich core in which the field might be generated by dynamo action (Ness et al., 1975; Stevenson, 1975; Gubbins, 1977b; Gault et al., 1977). However, there are no moment of inertia data to confirm the existence of a core. Another possible cause of Mercury's field is remanent magnetization of its outer layers (Stephenson, 1976; Sharpe and Strangway, 1976) but this requires a cold planet with unlikely high levels of magnetization. The field is not explainable by electromagnetic induction (Herbert et al., 1976).

The purpose of this paper is to suggest an explanation for the present-day existence or nonexistence of intrinsic magnetic fields of the terrestrial planets in terms of their compositions, structures and thermal histories. We assume that each of the terrestrial planets underwent primordial differentiation into a core and a mantle. We further assume that dynamo action requires either thermal or chemical convection in a fluid core or fluid outer core shell. Core evolution necessarily depends on the heat transport in the overlying mantle and no meaningful conclusions concerning the rate of cooling or freezing of a planetary core can be made without modeling the core and mantle as a coupled system. All terrestrial planetary mantles are likely to be undergoing thermal subsolidus convection (Schubert, 1979) and their thermal states are thus determined by their strongly temperature dependent rheologies and convective efficiencies in a self-regulatory manner (Tozer, 1965). For realistic rheologies, the resulting present-day deep mantle temperatures are almost certainly less than the melting temperature of pure iron. This expectation follows from the similarity of the melting temperatures for pure iron and major silicate phases at all relevant pressures (Basaltic Volcanism Study Project, 1981, Ch. IX; Stevenson, 1981; Brown and McQueen, 1982) together with the inference that any plausible mantle heat flow can be transported by subsolidus convection (Schubert, 1979). This appears to be true for Earth at least, independent of whether the mantle is layered, because the lower mantle is known from seismic data to be mostly solid, and its viscosity of  $10^{17}$ - $10^{18}$  m<sup>2</sup> s<sup>-1</sup>  $sec^{-1}$  (Peltier, 1981) can be achieved at a temperature of 0.8–0.9 of the melting point of major silicate phases or pure iron. Although the calculations presented in this paper indicate the possibility of a substantial temperature drop across a thermal boundary layer at the base of the mantle, this is insufficient to place the core temperature above the melting point of pure iron. It follows that the existence of partially fluid cores requires alloying constituents (e.g., sulfur or oxygen) which reduce the freezing points. These light alloying constituents in the cores of the terrestrial planets are also required on cosmochemical grounds (Basaltic Volcanism Study Project, 1981). The concentration of light constituent is constrained by a planet's average density, its mantle composition, and its core size.

If the cores of the terrestrial planets lack a substantial radioactive heat source such as potassium (Oversby and Ringwood, 1972; Ganguly and Kennedy, 1977) then the core convective motions necessary for a dynamo are either driven by secular cooling of an entirely fluid core or by inner core solidification. We will show that with secular cooling alone, the heat fluxes from the cores of all the terrestrial planets would probably be subadiabatic at present and dynamo action would have ceased more than a billion years ago. The absence of an inner core is then the most likely explanation for the lack of an intrinsic magnetic field for Venus and Mars. However, we cannot exclude models where the cores of Venus and Mars are almost completely frozen and we will present a model of Venus with a nearly solidified core. Complete solidification is unlikely because the self-regulated mantle temperature is likely to be higher than the iron alloy eutectic temperature (e.g., FeS eutectic) at present. The existence of intrinsic magnetic fields for Earth and Mercury follows naturally from our model as a consequence of continuing inner core growth.

The calculations assume subsolidus whole mantle convection parameterized by a simple Nusselt number-Rayleigh number relationship. This simple parameterization permits consideration of a wide range of models and parameter space and has previously been used with success by various authors (Sharpe and Peltier, 1978, 1979; Cassen et al., 1979; Stevenson and Turner, 1979; Schubert et al., 1979a,b, 1980; Davies, 1980; Richter and McKenzie, 1981; Schubert and Spohn, 1981; Cook and Turcotte, 1981; Spohn and Schubert, 1982a). The following section gives a mathematical description of our model. It is followed by a discussion of relevant parameter values and by a section presenting the model results.



FIG. 1. Temperature T(r) and core liquidus temperature  $T_m(r)$  as a function of radial distance r from the planet's center. The radius of the inner core is  $R_i$ ,  $R_c$  is the radius of the core, and  $R_p$  is the planet's radius. Temperature rises by  $\Delta T_s$  from the surface temperature  $T_s$  to the upper mantle temperature  $T_u$  across the surface boundary layer of thickness  $\delta_s$ . It rises by  $\Delta T_c$ from the lower mantle temperature  $T_i$  to the coremantle boundary temperature  $T_{cm}$  across the bottom boundary layer of the mantle which is of thickness  $\delta_c$ .  $T_{miso}$  is the liquidus temperature at the inner core-outer core boundary.

The final section contains a discussion of these results and our conclusions.

### **II. THE MODEL**

The model consists of a spherical shell, the planetary mantle, surrounding a concentric spherical core. The mantle material has average density  $\rho_{\rm m}$ , an average heat capacity  $C_{\rm m}$ , and a thermal conductivity k. The core is fluid except for the possibility of a concentric, spherical, solid inner core. We assume a constant average core density  $\rho_{\rm c}$  and an average heat capacity  $C_{\rm c}$ . The outer radius of the mantle is the planet's equatorial radius  $R_{\rm p}$ , and the inner mantle radius is the core radius  $R_c$ . For a crude two-layer model,  $R_c$  can be obtained from the planetary mass,  $R_p$ ,  $\rho_m$ , and  $\rho_c$ . The radius of an inner core-outer core boundary is denoted by  $R_i$ .

Figure 1 schematically represents a model temperature profile for a terrestrial planet. Throughout this work, whole mantle convection has been assumed and no allowance has been made for mantle phase changes. Although layered mantle convection is possible, the models of Spohn and Schubert (1982a) for Earth show that it is difficult to reconcile with current viscosity estimates and likely heat source distributions. If Earth or other terrestrial planets have chemically stratified mantles, then the models presented here are inapplicable. The omission of phase change effects is less of a problem since (as we elaborate below) the general conclusions are not contingent on precise evaluation of deep planet temperatures. Because the planet's mantle is undergoing vigorous, subsolidus convection, there are thermal boundary layers at the top and bottom of the mantle. The thickness of a boundary layer is denoted by  $\delta$ ; the subscripts s and c refer to the boundary layer next to the surface and core, respectively. We assume that temperature varies linearly with depth or radius in the boundary layers and  $\Delta T$  is the temperature drop across a boundary layer. Mantle temperature  $T_m$  increases from the "surface" temperature  $T_s$  to the upper mantle temperature  $T_{\rm u}$  across the upper boundary layer and from  $T_1$  to  $T_{cm}$  across the lower boundary layer. The "surface" temperature  $T_s$ may refer to either the physical surface of the planet (as in the case of Earth) or the level at which convection ceases (as in the case of one-plate planets: Mercury, Mars, possibly Venus).  $T_{cm}$  is the temperature at the core-mantle boundary. Temperature increases adiabatically from  $T_{\mu}$  to  $T_{l}$  across the interior of the mantle convection system. We assume that the above temperatures are spherically averaged quantities.

The kinematic viscosity  $\nu$  of the mantle is assumed to be related to the absolute upper mantle temperature  $T_u$  by

$$\nu = \nu_0 \exp\left(\frac{A}{T_u}\right), \qquad (1)$$

where  $\nu_0$  and A are constants. It is convenient to determine the viscosity in terms of the upper mantle temperature rather than in terms of the average mantle temperature. In the latter case, the parameter A would take different values from planet to planet since the reference pressure would change with the planet's size. The decay of radioactive isotopes in the planetary mantles is assumed to produce heat at the rate Q per unit volume and per unit time. The heat sources diminish with time according to

$$Q = Q_0 \exp(-\lambda t), \qquad (2)$$

where  $Q_0$  is the initial heat source density and  $\lambda$  is the average decay constant. A more precise parameterization of the radiogenic heat production would not significantly change any of the models or conclusions presented here because it would only differ significantly at early times ( $t \leq 10^9$ years). We assume vigorously convecting planets which start their evolution at high temperatures and therefore do not retain memory of the details of their early evolution (Schubert et al., 1980). This should be contrasted with "cold start" models (e.g., Toksöz et al., 1978; Siegfried and Soloman, 1974) where the details of isotopic composition may play a significant role.

Temperature increases adiabatically across the fluid outer core from  $T_{\rm cm}$  to  $T_{\rm mio}$ , the liquidus of the core alloy, if an inner core exists (as in Fig. 1). Nonadiabatic temperature differences or boundary layers are negligible in a convective outer core because of the low viscosity. We assume that the inner core consists of pure iron and the outer core contains a light alloying constituent. However, we neglect inner core-outer core density differences for the purposes of estimating pressures (but not for the purposes of estimating gravitational energy releases). The liquidus temperature  $T_{\rm m}$  of the core alloy is expressed as a quadratic in the pressure P(r):

$$T_{\rm m}(r) = T_{\rm m0}(1 - \alpha_{\rm c} x)$$

$$(1 + T_{\rm m1} P(r) + T_{\rm m2} P^2(r)), \quad (3)$$

where  $T_{m0}$ ,  $T_{m1}$ ,  $T_{m2}$  are constants, r is the radial distance from the planet's center, x is the mass fraction of light alloying constituent ( $x \ll 1$  is assumed), and  $\alpha_c$  is a constant. The models assumed  $\alpha_c = 2$ , appropriate to sulfur (Usselman, 1975a,b) but this is not crucial to the validity of the model, as we discuss later. The parameter choices in (3) are guided by Lindemann's law (Stacey, 1977a) and are therefore related to the parameters that enter in the core adiabat,

$$T_{\rm c}(r) = T_{\rm cm} \left\{ \frac{1 + T_{\rm a1} P(r) + T_{\rm a2} P^2(r)}{1 + T_{\rm a1} P_{\rm cm} + T_{\rm a2} P_{\rm cm}^2} \right\}, \quad (4)$$

where  $T_c$  denotes the temperature in the outer core,  $P_{cm}$  is the pressure at the coremantle boundary, and  $T_{a1}$  and  $T_{a2}$  are constants. These constants are determined by choices of Grüneisen's  $\gamma$  for the outer core and are discussed in the next section.

The simultaneous solution of (3) and (4) gives the pressure  $P_{i0}$  at the inner coreouter core boundary. The radius of the inner core is then obtained by assuming that the acceleration of gravity is  $rg/R_c$  (where g is the *surface* value), and is given by

$$R_{\rm i} = \{2(P_{\rm c} - P_{\rm i0})R_{\rm c}/\rho_{\rm c}g\}^{1/2}, \qquad (5)$$

where  $P_c$  is the pressure at the planet's center. (This approximation works well for Earth.) The mass *m* of the inner core is then

$$m = \frac{4}{3}\pi R_i^3 \rho_c. \tag{6}$$

Initially, the whole core is superliquidus and  $R_i = 0$ . As the planet cools, inner core nucleation begins as the liquidus temperature is reached at the planet's center. (The liquidus curve is always steeper than the adiabat because we choose  $\gamma > \frac{2}{3}$ ; Stevenson, 1980). With further cooling, the inner core grows at the expense of the outer core. At the same time, the liquidus temperature profile of the outer core will be lowered because the concentration of light constituent increases. By conservation of light constituent mass

$$x = x_0 \cdot \frac{M}{M - m} = \frac{x_0 R_c^3}{R_c^3 - R_1^3}, \quad (7)$$

where *M* is the mass of the whole core and  $x_0$  is the initial concentration of light constituent. An increase of inner core mass by  $\delta m$  in time  $\delta t$  releases a quantity of energy  $(L + E_G)\delta m$ , where *L* is the latent heat of solidification and  $E_G$  is the gravitational energy made available (and lost eventually as heat), per unit mass of inner core material. The energy  $E_G$  arises by the exclusion of the light constituent from the inner core.

The energy balance equations for mantle and core are

$$\frac{4}{3}\pi(R_{\rm p}{}^{3}-R_{\rm c}{}^{3})\left\{Q-\rho_{\rm m}C_{\rm m}\frac{d\langle T_{\rm m}\rangle}{dt}\right\}$$
$$=4\pi\{R_{\rm p}{}^{2}F_{\rm s}-R_{\rm c}{}^{2}F_{\rm c}\},\quad(8)$$

$$-\frac{4}{3}\pi R_{\rm c}^{3}\rho_{\rm c}C_{\rm c}\frac{d\langle T_{\rm c}\rangle}{dt}$$
$$+(L+E_{\rm G})\frac{dm}{dt}=4\pi R_{\rm c}^{2}F_{\rm c},\quad(9)$$

where  $\langle T_m \rangle$  is the average mantle temperature

$$\langle T_{\rm m} \rangle = \frac{1}{\frac{4}{3}\pi (R_{\rm p}^{3} - R_{\rm c}^{3})} \int_{R_{\rm c}}^{R_{\rm p}} 4\pi r^{2} T_{\rm m}(r) dr,$$
(10)

t is time,  $F_s$  is the surface heat flux, and  $F_c$  is the heat flux from the core. We relate  $\langle T_m \rangle$  to the upper mantle temperature  $T_u$  by

$$\langle T_{\rm m} \rangle = \eta_{\rm m} T_{\rm u} \tag{11}$$

where  $\eta_m$  is a constant. In (9),  $\langle T_c \rangle$  is the average temperature in the outer core

$$\langle T_{\rm c} \rangle = \frac{1}{\frac{4}{3}\pi (R_{\rm c}^3 - R_{\rm i}^3)} \int_{R_{\rm i}}^{R_{\rm c}} 4\pi r^2 T_{\rm c}(r) dr, \quad (12)$$

with  $T_c(r)$  given by (4). Evaluation of (12) shows that

$$\langle T_{\rm c} \rangle = \eta_{\rm c} T_{\rm cm} \tag{13}$$

with  $\eta_c$  a constant. Using (11) and (13) together with

$$\frac{dm}{dt} = 4\pi R_i^2 \rho_c \frac{dR_i}{dt}$$
$$= 4\pi R_i^2 \rho_c \frac{dR_i}{dT_{\rm cm}} \frac{dT_{\rm cm}}{dt}, \quad (14)$$

(8) and (9) become

$$\frac{4}{3}\pi(R_{\rm p}{}^{3}-R_{\rm c}{}^{3})\left\{Q-\rho_{\rm m}C_{\rm m}\eta_{\rm m}\frac{dT_{\rm u}}{dt}\right\}$$
$$=4\pi\{R_{\rm p}{}^{2}F_{\rm s}-R_{\rm c}{}^{2}F_{\rm c}\},\quad(15)$$

$$\left\{ (L + E_{\rm G}) 4\pi R_{\rm i}^2 \rho_{\rm c} \frac{dR_{\rm i}}{dT_{\rm cm}} - \frac{4}{3} \pi R_{\rm c}^3 \rho_{\rm c} C_{\rm c} \eta_{\rm c} \right\}$$
$$\cdot \frac{dT_{\rm cm}}{dt} = 4\pi R_{\rm c}^2 F_{\rm c}. \quad (16)$$

 $R_{\rm i}$  as a function of  $T_{\rm cm}$  is obtained from the simultaneous solution of (3)–(5).  $dR_{\rm i}/dT_{\rm cm}$  is obtained by differentiating the resulting equation and is dependent on known quantities only.

The heat fluxes  $F_s$  and  $F_c$  are given in terms of the temperature drops  $\Delta T$  across the thermal boundary layers and their thicknesses  $\delta$  by

$$F = \frac{k\Delta T}{\delta}.$$
 (17)

If the boundary layer thicknesses are globally determined (Turcotte and Oxburgh, 1967) then

$$\delta = (R_{\rm p} - R_{\rm c}) \left(\frac{Ra_{\rm cr}}{Ra}\right)^{\beta}, \qquad (18)$$

where  $\beta$  is a constant, and  $Ra_{cr}$  is approximately the critical Rayleigh number for the onset of convection in the mantle shell (but should more correctly be thought of as an empirical parameter chosen to be consistent with numerical and laboratory experiments). The Rayleigh number Ra is defined by

$$Ra = \frac{g\alpha(\Delta T_{\rm s} + \Delta T_{\rm c})(R_{\rm p} - R_{\rm c})^3}{\nu K}, \quad (19)$$

where  $\alpha$  is the volumetric coefficient of thermal expansion and K is the average thermal diffusivity in the mantle.

Equations (17)-(19) have been previously used by numerous authors to calculate thermal histories. Schubert *et al.* (1979a) provide a detailed discussion of their applicability. Equation (18) is basically valid for constant viscosity fluids; it assigns the same thickness to the thermal boundary layers at the top and the bottom of the convecting shell. However, because of the strongly temperature-dependent viscosity of the mantle it is possible that the lower boundary layer is thinner, on the average, than the upper boundary layer (Daly, 1980; Nataf and Richter, 1982). The lower boundary layer might also be thinned by the ejection of plumes and thermals as a consequence of buoyancy instability enhanced by a reduction in viscosity (Howard, 1966; Richter, 1978; Yuen and Peltier, 1980); it might even vanish altogether, on the average, as a result of this process. We model the reduction in boundary layer thickness at the core-mantle boundary by determining its thickness locally whenever the heat flux from the core is sufficiently large. The experiments of Booker and Stengel (1978) suggest that the local critical Rayleigh number for the breakdown of the boundary layer is

$$Ra_{\rm crb} = \frac{g\alpha\Delta T_{\rm c}\delta_{\rm c}^{3}}{\nu_{\rm c}K} \approx 2 \times 10^{3}.$$
 (20)

Richter (1978) finds that  $v_c$  should be based on the average temperature within the boundary layer. Hence,

$$\nu_{\rm c} \equiv \nu_0 \exp\left(\frac{A}{T_1 + \frac{\Delta T_{\rm c}}{2}}\right).$$
 (21)

In most models we have used Eq. (20) instead of (18) to calculate  $\delta_c$  whenever (20) gave a smaller thickness.

The integration of the system of ordinary differential Eqs. (14)-(16) with respect to time determines the thermal history of a planet in terms of its upper mantle temperature  $T_u$ , its core-mantle boundary temperature  $T_{cm}$ , and the radius of the inner core. The boundary and initial conditions for (14)-(16) are

$$T(R_{\rm p}) = T_{\rm s}, \qquad (22)$$

$$T_{\rm u}(t=0) = T_{\rm uo},$$
 (23)

TABLE I	
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PARAMETER VALUES FOR CORE Evolution Models of the Terrestrial Planets

Parameter	Value		
α	$2 \times 10^{-5} ^{\circ}\text{K}^{-1}$		
k	4.0 W m <sup>-1</sup> °K <sup>-1</sup>		
K	$10^{-6} \text{ m}^2 \text{ sec}^{-1}$		
$\rho_{\rm m}C_{\rm m}$	$4.0 \times 10^{6} \text{ J m}^{-3} ^{\circ}\text{K}^{-1}$		
$\rho_{\rm m}C_{\rm m}/\rho_{\rm s}C_{\rm s}$	1		
$Q_0$	$1.7 \times 10^{-7}$ W m <sup>3</sup>		
λ	$1.38 \times 10^{-17} \text{ sec}^{-1}$		
Α	5.2 × 10 <sup>4</sup> °K		
$\nu_0$	$4.0 \times 10^3 \text{ m}^2 \text{ sec}^{-1}$		
Raci	$5.0 \times 10^{2}$		
β	0.3		

$$T_{\rm cm}(t=0) = T_{\rm co},$$
 (24)

$$R_{i}(t=0) = 0.$$
 (25)

## III. MODEL PARAMETERS

It is reasonable to assume that certain of the model parameters are approximately independent of the size and mass of the planet and particular core chemistry; we adopt the Earth values of these parameters for all the planets. Table I lists these parameters and values we have assigned them throughout the study. The list includes the average mantle values of the thermal expansion coefficient  $\alpha$ , thermal conductivity k, the thermal diffusivity K, the product of mantle density and heat capacity  $\rho_m C_m$ , the ratio  $\rho_{\rm m} C_{\rm m} / \rho_{\rm c} C_{\rm c}$ , the initial radioactive heat generation rate per unit volume  $Q_0$ , the mean decay constant  $\lambda$ , and the viscosity parameters A and  $\nu_0$ . The latter two parameters are constrained to satisfy the Earth's present-day mantle viscosity of order 10<sup>17</sup>  $m^2 sec^{-1}$  (Cathles, 1975; Peltier, 1981) in the model calculations.  $Q_0$  has been adjusted so that Earth's present-day surface heat flux of ~60 mW m<sup>-2</sup> (Sclater et al., 1980; Turcotte and Schubert, 1982) is obtained. With a chondritic choice for  $\lambda$ , the present value of the mantle heat generation rate per unit volume is found to be  $2.5 \times 10^{-8}$  W m<sup>-3</sup>.

PARAMETER VALUES FOR CORE EVOLUTION MODELS OF THE TERRESTRIAL PLANETS

	Mercury	Venus	Earth	Mars	Units
R,	2440.	6051.	6371.	3389.	× 10 <sup>3</sup> m
8	3.8	9.0	10.	3.7	m sec <sup>2</sup>
Ť,	1073.	730.	293.	1073.	°К
$T_{m0}$	1880.	1960.	Varied	1880.	°К
T <sub>ml</sub>	1.36	6.14	6.14	1.36	°K TPa <sup>†</sup>
T <sub>m2</sub>	-6.2	-4.5	-4.5	-6.2	°K TPa <sup>2</sup>
Tai	8.00	3.96	3.96	8.00	°К ТРач
T.,	-3.9	-3.3	-3.3	-3.9	°К ТРа ?
η <u>,</u>	1.00	Varied	1.30	1.00	
n	1.10	Varied	1.20	1.10	

This is 75% of the heat generation rate that would be in equilibrium with the presentday mantle heat flux of 60 mW  $m^{-2}$  and is comparable to the heat production rate per unit volume of a potassium-depleted chondritic Earth mantle of  $2.6 \times 10^{-8}$  W m<sup>-3</sup> (Stacev, 1977a). It has been noted before that a significant part of the Earth's present-day surface heat flow should be due to secular cooling (Sharpe and Peltier, 1978; Schubert et al., 1980; Davies, 1980; Spohn and Schubert, 1982). Of course,  $Q_0$ and  $\lambda$  may vary from planet to planet and we did construct models with some variation in  $Q_0$ . However, we have chosen to keep the problem (and the number of models described) to manageable proportions by adopting Earth values in most models. The consequences of different choices are discussed in the final section. Table I also gives the values assigned to  $Ra_{cr}$  and  $\beta$ . For a discussion of their values applicable to planetary thermal history calculations, see Schubert et al. (1979a).

Table II lists the values of parameters which are specific to individual planets and which have been retained throughout most of our study. Equatorial radii are given, for instance, by Cook (1980). The surface temperatures for the models of Earth and Venus are the actual surface temperatures. Mars and Mercury probably have very thick lithospheres (Schubert *et al.*, 1979a) which do not participate in mantle convection. The temperature difference across the bulk of a thick, intact lithosphere does not help to drive mantle convection. Therefore, we have chosen  $T_s$  for Mercury and Mars to be the approximate temperature at which silicate rock may undergo sufficient flow to participate in the convection. Venus may also be a one-plate planet (Phillips *et al.*, 1981) but its surface temperature is already high.

The values of the coefficients of the polynomials (3) and (4) that approximate the core liquidus and adiabat are different because of the diverse pressures encountered in the cores of the terrestrial planets. Leastsquares fitting of Stacey's (1977b) liquidus for the Earth's core gives  $T_{m0} = 2060^{\circ}$ K if x = 0.1. We have allowed  $T_{m0}$  to vary slightly in our models so as to reproduce the correct inner core size at the present day. Typically,  $T_{\rm m0} \simeq 1960^{\circ} {\rm K}$  is needed, depending on the choice of  $L + E_G$ . For Venus, we retain the same choice as for the Earth. For Mercury and Mars,  $T_{m0} = 1880^{\circ}$ K which is close to the 1-bar melting point of pure iron. The parameterization for Mercury and Mars essentially reproduces the high-pressure data of Liu and Bassett (1975). The choices of  $T_{a1}$  and  $T_{a2}$  are for the Grüneisen  $\gamma$  of Stacey (1977b).

The values of the constants  $\eta_m$  and  $\eta_c$ which relate the average mantle and core temperatures to the upper mantle temperature  $T_u$  and the temperature  $T_{cm}$  at the coremantle boundary are given next in Table II. For the small planets Mars and Mercury, the adiabatic temperature difference across the mantle is small and  $\eta_m$  is taken to be unity. For these planets, evaluation of (12) and (4) for reasonable core radii (see below) gives  $\eta_c \approx 1.1$ . For Earth we take  $\eta_m = 1.30$ in accordance with Stacey's (1977b) mantle geotherm and  $\eta_c = 1.20$ . The values of  $\eta_m$ and  $\eta_c$  for Venus are close to those for Earth but have been varied according to the different models of core chemistry considered.

The parameters that have been varied during our modeling include the core densi-

ties of Mercury, Venus, and Mars, which have been adjusted for the concentration of light constituent(s). One Venus model also considers a 2% lower average mantle density with a 100- to 150-km-thick crust (Anderson, 1980) to account for the ~2% lower intrinsic density of Venus as compared with Earth. The specific energy release  $L + E_G$ upon freezeout of the inner core has also been varied in the range  $2.5 \times 10^5$  to  $2 \times 10^6$ J kg<sup>-1</sup> and is usually dominated by  $E_G$ , at least for Earth and Venus. The latent heat is given by

$$L = T_{\rm m} S_{\rm m} \tag{26}$$

where  $S_m$  is the entropy of melting. At low pressures, L is  $2.5 \times 10^5$  J kg<sup>-1</sup> for pure iron. The melting point of iron increases by about a factor of 3 in going from low pressure to 300 GPa (Brown and McQueen, 1980, 1982), while  $S_m$  decreases only slightly. Accordingly, L is around  $5-8 \times 10^5$ J kg<sup>-1</sup> at 300 GPa. The gravitational energy release depends on core size and composition and can be as large as  $2 \times 10^6$  J kg<sup>-1</sup> (Loper, 1978). The effective L and  $E_{\rm G}$  are modified by the gravitational work done as the core radius changes during inner core freezeout (Häge and Müller, 1979; Müller and Häge, 1979) but these are minor considerations compared with other uncertainties in the calculations. The initial concentration of light constituent  $x_0$  is taken to be 0.1 for Earth, consistent with sulfur (Ahrens, 1979), and it varies from  $10^{-3}$  to 0.1 for Venus, from 0.1 to 0.25 for Mars, and from 0.01 to 0.05 for Mercury.

## **IV. RESULTS**

We have solved Eqs. (14) through (16) numerically using a Runge-Kutta predictor-corrector scheme and have monitored the evolution of the model planets' cores and mantles. In particular, we have watched the onset and continuation of inner core growth, the core and surface heat flows, the upper mantle and core-mantle boundary temperatures, the mantle viscosity, and the mantle Rayleigh number. We

have explored the parameter space of successful models that explain the observed magnetic properties of the terrestrial planets. A successful model of Earth has a present-day surface heat flow of about 60 mW m<sup>-2</sup>, a kinematic mantle viscosity of order 1017 m<sup>2</sup> sec<sup>-1</sup> and an inner core radius of ~1215 km. Also, to ensure dynamo action, the Earth's outer core must convect. If thermal convection drives the dynamo, then the core heat flux must be larger than  $F_{\text{cond}}$ , the heat flux conducted along the core adiabat. For  $k \simeq 40$  W m<sup>-1</sup> °K<sup>-1</sup>, indicated by liquid state calculations and application of the Wiedemann-Franz relation (Stevenson, 1981),  $F_{cond}$  is 10 to 20 mW m<sup>-2</sup>. We have chosen a nominal value of 15 mW m<sup>-2</sup> for the Earth, but we recognize that the cessation of thermal convection, if it occurred, is not likely to be simultaneous throughout the core since both the actual flux and the conductive flux vary with radial position (see, for example, Gubbins, 1976). If chemical convection drives the dynamo, then the energy release per unit area from inner core growth must exceed  $\sim \varepsilon F_{\text{cond}}$ , where  $\varepsilon$  is the Carnot efficiency ~0.1 (Gubbins, 1977a; Stevenson, 1983). In practice, this criterion is readily satisfied if the inner core is growing.

For Mercury, a "successful" model is one in which the outer core still convects, while for Venus and Mars, models without convection are desirable. The successful models for Earth serve as a guide for finding successful models for the other bodies.

# Earth

Table III lists six successful Earth models. All Earth models have an outer core of radius 3485 km, a core density  $\rho_c$  of  $1.3 \times 10^4$  kg m<sup>-3</sup>, and an initial sulfur concentration  $x_0 = 0.1$ . The pressure at the Earth's center is 0.36 TPa and the pressure at the core-mantle boundary is 0.14 TPa, in accordance with Dziewonski *et al.* (1975). Models E1 and E2 are our nominal models with all other parameter values as discussed in the previous section. These two

	EI	E2	E3	E4	E5	E6
$L + E_{\rm G} [10^6  \rm J  kg^{-1}]$	1	2	1	2		i
<i>T</i> <sub>m0</sub> (°K)	1950	1980	1980	2030	1600	1660
γ	$\sim \rho^{-0.6}$	$\sim  ho^{-0.6}$	$\sim  ho^{-0.6}$	$\sim  ho$ 06	1.7	$\sim \rho^{-1}$
δ <sub>c</sub>	(20)	(20)	(18)	(18)	(20)	(20)
Onset (by)	2.7	2.3	2.9	2.4	2.9	3.0
<i>R</i> <sub>i</sub> (km)	1234.	1207.	1215.	1229.	1192.	1185.
$F_{\rm c} ({\rm mW}{\rm m}^{-2})$	18.6	24.4	17.4	21.2	17.1	17.2
F, (mW m <sup>-2</sup> )	62.7	64.1	63.1	64.0	62.5	62.7
<i>T</i> <sub>u</sub> (°K)	1648.	1650.	1648.	1648.	1647.	1647
$T_{\rm cm}$ (°K)	2960.	3010.	3010.	3085.	3004.	3005.
T <sub>mio</sub>	3956.	4017	4022	4116	3254	3376
$\frac{dR_{i}}{dt}$ (mMa <sup>-1</sup> )	0.25	0.20	0.23	0.17	0.23	0.25
$\nu_{\rm m} ({\rm m}^2{\rm sec}^{-1})$	$2 \times 10^{17}$					
Ra	$6 \times 10^{8}$					

TABLE III

PARAMETERS AND RESULTS OF SUCCESSFUL EARTH MODELS

Note. The first two rows give the values of the core thermal parameters  $L + E_G$  and  $T_{m0}$ . The following two rows indicate the functional dependence of the Grüneisen parameter  $\gamma$  on core density and the relation used to calculate the thickness of the core-mantle thermal boundary layer. Next, the time for the onset of inner core freezing is listed. The following entries give present-day values of the inner core radius  $R_i$ , the core heat flux  $F_c$ , the surface heat flux  $F_s$ , the upper mantle temperature  $T_u$ , the core-mantle boundary temperature  $T_{cm}$ , the temperature at the inner core-outer core boundary  $T_{m0}$ , the rate of inner-core growth  $dR_i/dt$ , the mantle viscosity  $\nu_m$ , and the mantle Rayleigh number Ra.

models differ in the choice of  $L + E_G$  (10<sup>6</sup> J kg<sup>-1</sup> for E1 and 2 × 10<sup>6</sup> for E2) and in  $T_{m0}$ , which has been adjusted to give the correct size of the present inner core. Models E3 and E4 differ from E1 and E2, respectively, in that they use (18) instead of (20) to calculate the thickness of the core-mantle thermal boundary layer. In E5 and E6, Eq. (20) is used, as in the nominal models, but the melting and adiabatic profiles in the core are modified. In E5, a constant  $\gamma = 1.7$  is used (compatible with liquid state models; Stevenson, 1981) while in E6,  $\gamma \propto \rho^{-1}$ .

Inner core growth for all models begins after 2.3 to 3.0 by of thermal evolution and results in inner core radii of 1185–1234 km after 4.5 by. (A better fit to the observed inner core radius could have been obtained by fine tuning  $T_{m0.}$ ) Models E2 and E4 take about 600 my longer than the other models to freeze the present-day inner core because twice the amount of energy per unit mass of inner core,  $L + E_G$ , has to be removed from the outer core. At present, the rate of inner core freezing in these models is still some tens of percent less than in the other models.

Inner core radius as a function of time is presented in Fig. 2 for the nominal models. Except near the onset of inner core growth, the radius of the inner core is seen to increase proportionately with the one-fourth power of time elapsed since onset of inner core growth. This is a purely empirical result and the one-fourth power has no special significance (one would expect onethird if the inner core mass were increasing uniformly with time).

Figures 3-5 show core heat flux versus time for E1-E4 and illustrate these models in more detail. The core heat flux initially drops rapidly with time because of the relatively low viscosity of the Archean mantle. It drops faster for models E1 and E2, which cool more effectively than E3 and E4 (Figs. 4 and 5) because of a larger heat transfer across the destabilized and thinned coremantle thermal boundary layers of E1 and



FIG. 2. Radius  $R_t$  of the Earth's inner core from models E1 and E2 as a function of  $(t - t_0)^{0.25}$ , where t is time and  $t_0$  is the time of onset of inner core freezing. The curve parameter is the specific energy release upon inner core freezing in 10<sup>6</sup> J kg<sup>-1</sup>. Except near  $t = t_0$ ,  $R_{tc}$  is approximately proportional to  $(t - t_0)^{0.25}$ .

E2. Without inner core freezing, the heat flux from the core would probably fall below the conductive heat flux along the adiabat and core convection would have ceased



FIG. 3. Thermal histories of the cores of Earth models E1 and E2 and Venus model V1. The curve parameter for the Earth models is the specific energy release upon inner core freezing in J kg<sup>-1</sup>. The dashdot line marks the conducted heat flux along the core adiabat. Without inner core solidification, larger heat fluxes indicate thermal convection in the core. Smaller heat fluxes are then indicative of cessation of thermal convection and dynamo generation. With inner core growth, chemical buoyancy helps to drive convection and the core heat flux may be sub- or superadiabatic.



FIG. 4. Thermal histories of the cores of Earth models E3 and E4 (solid lines) compared to thermal histories of E1 and E2 (dashed lines). Curve parameter is the specific energy release upon inner core freezing in  $10^6$  J kg<sup>-1</sup>. Models E3 and E4 have upper and lower thermal boundary layers of the same thickness in the mantle while E1 and E2 have thinned lower boundary layers whose thicknesses are based on a local stability criterion (20). Models E1 and E2 are much more effective at heat removal from the core than E3 and E4.

after 3 to 4 by of thermal evolution. The Earth's magnetic field would have died (free decay time  $\sim 10^4$  years) at that time. With inner core growth, the rate of core cooling decreases markedly as a consequence of the coupling of heat production by core freeze out to cooling. The core heat flux tends to a plateau of 18 to 25 mW m<sup>-2</sup> for E1 and E2, depending on the energy release per unit mass of inner core. For



FIG. 5. Thermal histories of the cores of E1 and E3 showing early time behavior. For further explanation see Fig. 4.

	Earth-like model			Anderson model			Iron-rich model		
	VI	V2	V3	V4	V5	V6	V7	V8	V9
<i>R</i> <sub>s</sub> (km)	3110	3110	3110	3230	3230	3230	2890	2890	2890
$\rho_{\rm c} [10^3  \rm kg  m^{-3}]$	12.5	12.5	12.5	12.5	12.5	12.5	13.0	13.0	13.0
X <sub>0</sub>	0.10	0.08	0.10	0.11	0.10	0.06	10 3	10-2	10 2
P <sub>c</sub> (GPa)	290.	<b>290</b> .	290.	<b>295</b> .	295.	295.	310.	310.	310.
P <sub>cm</sub> (GPa)	130.	130.	130.	120.	120.	120.	145.	145.	145.
η <sub>m</sub>	1.30	1.30	1.30	1.27	1.27	1.27	1.32	1.32	1.32
ης	1.20	1.20	1.20	1.15	1.15	1.15	1.15	1.15	1.15
$L + E_{\rm G} (10^{\circ}  {\rm J  kg^{-1}})$	10	10	10	10	10	10	5	5	10
A (104 °K)	5.2	5.2	4.8	5.2	5.2	5.2	5.2	5.2	5.2
Onset (by)	4.5	2.7	2.2	5.0	3.7	1.3	0.32	0.38	0.37
<i>R</i> , (km)	_	1154	1315	_	803	1840	2875	2724	2658
$F_{c} (mW m^{-2})$	10.7	18.6	20.7	10.5	12.5	30.0	14.2	20.9	42.7
F, (mW m <sup>-2</sup> )	59.9	62.1	61.6	58.9	59.3	63.0	63.4	65.0	68.3
$T_{\rm u}$ (°K)	1722.	1725.	1616.	1719.	1719.	1725.	1730.	1732.	1736.
$T_{\rm cm}$ (°K)	2933	3021	2839	2824	2855.	2990.	3059.	3117.	3247.
v (10 <sup>16</sup> m <sup>2</sup> sec <sup>-1</sup> )	5	5	3	6	6	5	5	4	4
Ra	$2 \times 10^{8}$	$2 \times 10^{8}$	$2 \times 10^{8}$	108	10 <sup>8</sup>	$2 \times 10^{8}$	$3 \times 10^{8}$	$3 \times 10^{8}$	$3 \times 10^{8}$

TABLE IV

PARAMETERS AND RESULTS FOR VENUS MODELS

Note. The Anderson models have a 100-km basaltic layer on top of the mantle. The iron-rich models have a low concentration of sulfur in the core. Onset refers to the time when inner core growth begins. The entries below onset are present-day values.

models E3 and E4, the core heat flux continues to decrease but at a lower rate (Fig. 4). Both chemical and thermal convection are available to drive the present dynamo, but the gravitational energy is most important because it is likely to be almost entirely available for dynamo generation (Gubbins, 1977a; also see Discussion).

# Venus

Three major types of models of Venus have been investigated and the results are presented in Table IV. For V1 and its variations V2 and V3, we assumed that the mantles and cores of Venus and Earth have essentially identical composition. The average core density is slightly lower for Venus  $(1.25 \times 10^3 \text{ kg m}^{-3})$  because of the  $\sim 20\%$  lower pressures in the core. A core radius of 3110 km is then required to reproduce the mass of  $4.87 \times 10^{24}$  kg. The central pressure in Venus for these models is chosen to be 290 GPa. (An accurate calculation of pressure, using seismically determined equations of state for each mantle layer and core, yielded 286 GPa for an Earth-like Venus.)

The second model type, represented by V4 and its variations V5 and V6, have larger cores (radius 3230 km) and a 100- to 170-km-thick basaltic crust, according to a suggestion by Anderson (1980). The central pressure is 295 GPa in these cases.

The third group of models (V7–V9 in Table IV) assumes a more iron-rich core than the Earth (i.e.,  $x_0 \leq 0.01$ ). With an Earthlike average crust and mantle, we get a core of radius 2890 km and a central pressure of 310 GPa. All models (V1–V9) have  $T_{m0} =$ 1960°K,  $\gamma \propto \rho^{-0.6}$  and assume an unstable boundary layer above the core-mantle interface, all consistent with the nominal Earth models.

Models with no core convection or dynamo at the present day are found to be possible for all three types of models. Earth-like model V1 has a present-day core heat flux of 10.7 mW m<sup>-2</sup>, which probably means the core is subadiabatic and nonconvective. The thermal history of V1 is shown in Fig. 3 and compared to successful Earth models. According to model V1, dynamo action ceased about 1.5 by ago when the core heat flux dropped below a value of



FIG. 6. Present temperatures and core melting temperatures in the interiors of Venus and Earth as functions of pressure from Earth model E1 and Venus model V1. The pressure of the core-mantle boundary is 130 GPa for Venus and 135 GPa for Earth. The temperatures in the cores of both planets are very similar but the pressure at Venus' center is below the pressure at the Earth's inner core boundary. According to model V1, Venus would be just beginning to nucleate a solid inner core.

15 mW m<sup>-2</sup> and the core became conductive and subadibatic. (The model ceases to be reliable beyond that point because adiabaticity continues to be assumed for the core. However, the model does indicate initiation of inner core freezeout after 4.5 by, about the present time. Implications of this are discussed in the final section.)

Figure 6 compares present-day temperatures in the cores of Earth and Venus from models E1 and V1. Although Venus is a smaller planet, the temperatures are very similar, partly because of Venus' high surface temperature. Venus' mantle is about 100°K hotter than Earth's mantle. The failure of V1 to nucleate an inner core is primarily due to the lower pressure at Venus' center. The surface heat flow for model V1 is 59.9 mW m<sup>-2</sup>, comparable to Earth. This is also the case for other Venus models.

Model V2 has a 20% lower initial concentration of light constituent, which gives rise to a higher core liquidus. Inner core freezeout occurs, not surprisingly, after 2.7 by and the present-day inner core radius is 1154 km. The core heat flow is higher, because of the energy release from inner core growth, and is 18.6 mW m<sup>-2</sup> after 4.5 by. Model V2 is very similar to the present Earth and would have a magnetic field.

Another way in which Venus could presently have inner core growth and magnetic field generation is by more efficient cooling. This occurs in model V3 which has a viscosity law (Eq. 1) in which  $A = 4.8 \times 10^4$ °K compared with the Earth-like value of  $5.2 \times 10^4$  °K. Inner core growth in V3 starts after 2.2 by of thermal evolution and results in a present-day inner core of 1315 km.

Models V4–V6, which have thick basaltic crusts, show similar results. The larger core does mean slightly higher pressures, however, and inner core nucleation is correspondingly easier if the concentration of light constituent is Earth-like or less. An increase in  $x_0$  from 0.1 to 0.11 is sufficient to keep the core from beginning to freeze after 4.5 by.

Models with very iron-rich cores (V7-V9) all have early inner core growth, because the initial liquidus is only slightly below the melting point of pure iron. Model V7, in which  $x_0 = 10^{-3}$ , has an almost completely frozen core at the present day, with only a 15-km-thick fluid layer remaining. Although convection persists, with the inner core growing at 3 km by<sup>-1</sup>, the magnetic Reynolds' number for this thin layer is probably too low for a dynamo. In V8 and V9, there is a higher concentration of the light constituent, and in V9 there is a higher energy release per unit inner core mass. These changes cause the fluid outer core to be thicker (e.g., 232 km after 4.5 by in V9). The large core heat flux of 42.7 mW m<sup>-2</sup> in V9 prevents the mantle from cooling enough to allow complete freezeout.

It is evident from our results that small changes in model parameters can result in completely fluid, nonconvecting cores without dynamo generation; convecting fluid outer cores with inner core growth and dynamo generation; and almost frozen cores with only thin outer core fluid shells remaining, probably with no dynamo. We argue in the concluding section that the first of these three outcomes is the most plausible state of Venus at present.

# Mars

For Mars, we consider an Earth-like model M1 with initial sulfur concentration  $x_0 = 0.1$ , a sulfur-rich model M2 with  $x_0 = 0.25$ , and a reduced radius model M3 which is also sulfur rich but allows for the thickness of the lithosphere in computing the effective depth of mantle convection. Table V describes these models.

In model M1, nucleation and growth of an inner core begins after 1.1 by of thermal evolution. The radius of the inner core is 952 km after 4.5 by, leaving a fluid outer core of 635 km in thickness. Convection in the outer core is maintained by chemical buoyancy, although the core heat flux is below the conductive value along an adiabat.

	M1	M2	M3
<i>R</i> <sub>p</sub> (km)	3389	3389	3200
$R_{\rm c}$ (km)	1589	1762	1762
$\rho_{\rm c} \ (10^3 \ {\rm kg \ m^{-3}})$	7.5	6.5	6.5
<i>x</i> <sub>0</sub>	0.1	0.25	0.25
P <sub>c</sub> (GPa)	47.0	41.0	41.0
P <sub>cm</sub> (GPa)	22.3	22.3	22.3
$L + E_{\rm G} (10^{5} {\rm J  kg^{-1}})$	5	5	5
Onset (by)	1.1	_	_
$R_{i}$ (km)	952	_	—
$F_{\rm c} ({\rm mW}{\rm m}^{-2})$	5.68	3.46	3.39
$F_{\rm s} ({\rm mW}{\rm m}^{-2})$	32.4	30.5	28.0
$T_{\mu}$ (°K)	1744	1740	1728
$T_{\rm cm}$ (°K)	1861	1816	1808
$\nu$ (10 <sup>16</sup> m <sup>2</sup> sec <sup>-1</sup> )	4	4	4
Ra	107	$8 \times 10^7$	$4 \times 10^7$

### TABLE V

PARAMETERS AND RESULTS OF MARS MODELS

*Note*. Onset marks the time when inner core growth begins. The entries below onset are present-day values.



FIG. 7. Thermal histories of Mars models M1 through M3. M1 has a solid inner core which nucleated at age 1.1 by. (Note the sharp change in the rate of core heat loss.) Models M2 and M3 have no inner cores. The sulfur contents of the models are marked. Model M1 could have a dynamo driven by chemically released buoyancy upon core freezing.

(This means that thermal convective transport is *downward* in the core; the chemical buoyancy is more than adequate to offset the slightly stable thermal state.) Figure 7 shows the thermal evolution and indicates that thermal convection might have ceased for a short period before inner core nucleation occurred. The sharp bend in the core heat flux versus time curve marks the onset of inner core freezing.

The sulfur-rich model M2 does not nucleate an inner core after 5 by. Thermal convection ceases after 1.2 by and there would be no subsequent dynamo action. Since any model with no inner core freezeout would have a similar thermal evolution, this model can be used to estimate the smallest initial sulfur fraction  $x_0$  for which no freezeout would occur after 4.5 by. The answer is  $x_0$ = 0.15.

Model M3 has an effective planetary radius of only 3200 km, 189 km less than Mars' equatorial radius. All other parameters are identical to M2. Mars is a one-plate planet with a thick lithosphere (Schubert *et al.*, 1979a; Sleep and Phillips, 1979) across which heat is conducted. Adoption of a lower effective planetary radius causes the mantle to cool slightly more than in M2 but

	Mel	Me2	Me3	Me4	Me5
$R_{p}$ (km)	2440	2440	2340	2440	2340
<i>R</i> <sub>c</sub> (km)	1840	1840	1840	1900	1900
$\rho_{\rm c}~(10^3~{\rm kg}~{\rm m}^{-3})$	8.6	8.6	8.6	8.2	8.2
<i>x</i> <sub>0</sub>	0.01	0.01	0.01	0.05	0.05
P <sub>c</sub> (GPa)	40.0	40.0	40.0	40.0	40.0
P <sub>cm</sub> (GPa)	10.0	10.0	10.0	10.0	10.0
$L + E_{\rm G} (10^5 {\rm J  kg^{-1}})$	2.5	5	2.5	5	5
Onset (10 <sup>2</sup> Ma)	2.3	2.3	2.2	6.0	5.80
<i>R</i> , (km)	1762	1749	1763	1408	1419.
<i>T</i> <sub>u</sub> (°K)	1634	1643	1618	1637	1622
$T_{\rm cm}$ (°K)	1794	1844	1788	1854	1845
F, [mW/m <sup>-2</sup> ]	16.26	17.81	14.65	17.25	15.63
$F_{\rm c} [{\rm mW}{\rm m}^{-2}]$	4.64	6.91	4.569	7.72	7.63
Ra	$7 \times 10^4$	$8 \times 10^4$	$3 \times 10^4$	$6 \times 10^4$	$2 \times 10^{4}$
$\nu (10^{16} \text{ m}^2 \text{ sec}^{-1})$	3	2	4	3	3

# TABLE VI Parameters and Results of Mercury Models

Note. Onset marks the time when inner core growth begins. The entries below onset are present-day values.

the model is otherwise very similar and no inner core freezeout occurs in 4.5 by. We argue in the concluding section that this kind of model, with a nonconvecting entirely fluid core, is the most likely explanation of the apparent lack of a Martian dynamo.

### Mercury

The main differences among the models of Mercury (Me1 through Me5 in Table VI) that we investigated are in the concentration of light constituent, which affects the liquidus, the core density, and the core size. We have also varied the energy release upon core freezing and the effective radius of the planet. Model Me1 has a core of nearly pure iron (initial sulfur content of only 1%), a core density of 8.6  $\times$  10<sup>3</sup> kg m<sup>-3</sup>, and a core radius of 1840 km. The energy release per unit mass upon core freezing is  $2.5 \times 10^5$  J kg<sup>-1</sup>, appropriate to the latent heat of iron (gravitational energy release being much smaller in Mercury than in Earth). Model Me2 differs from Me1 only in that the energy release per unit mass is doubled. Model Me3 is like Me1 but has a reduced radius of 2340 km, to allow for the

thick lithosphere, which should be excluded in the evaluation of the mantle Rayleigh number. Models Me4 and Me5 have an initial sulfur content of 5%, an average core density of  $8.2 \times 10^3$  kg m<sup>-3</sup>, and a core radius of 1900 km. Model Me5 has a reduced effective planetary radius of 2340 km.

Inner core nucleation and growth starts early in all our Mercury models. For Mel through Me3, inner core growth sets in after 230 my of planetary evolution; for models Me4 and Me5 it is delayed until 600 my. The present day inner core radius is about 1750 km for the very iron-rich models Me1-Me3, leaving only an 80- to 90-kmthick outer fluid shell. If five times more light constituent is allowed in the core alloy then this fluid shell is currently 480- to 490km-thick (Me4-Me5). Figure 8 gives the thermal histories of the outer core as a function of time for these five models. All models derive between 30 and 40% of their present-day heat output from secular cooling and gravitational energy release. The remaining energy is from mantle radiogenic heating.

Outer core convection persists in all



FIG. 8. Thermal histories for Mercury models Mel through Me5. The sulfur contents of the models are marked. All models have inner cores and may presently generate magnetic fields. The dynamos are driven by chemically released buoyancy upon inner core freezing.

these models and is maintained by chemical buoyancy. The heat flux from the core drops below the conductive heat flux along the adiabat after 2.3 by for models Me1 and Me2, after 3 by for Me4 and Me5, and after 3.4 by for Me2. However, chemical buoyancy exceeds the stabilizing effect of the subadiabatic heat flow. The minimum fluid shell needed for dynamo action is not known, but models Me4 and Me5, and possibly intermediate models (e.g.,  $x_0 = 0.03$ ) are likely to have dynamo generation.

## V. SUMMARY AND DISCUSSION

The thermal history of a terrestrial planetary core is governed by the ability of the mantle to cool the core and is greatly modified if the mantle allows the core to cool below the liquidus temperature of the core alloy. The liquidus temperature depends on core chemistry and pressures. The history of the magnetic field is tied to the thermal history of the core (Hewitt *et al.*, 1977; Gubbins *et al.*, 1979) and is therefore intimately related to mantle convection, core chemistry, and core pressures.

The translation of these general conclusions into specific statements about planets is fraught with difficulty because many of the important parameters are poorly known. We have presented a large number of models to encompass a range of possibilities. It is desirable to step back from the specifics and the details and try to extract the essential features. We do this by first summarizing the assumptions of our models. We then pose a number of potential criticisms and questions, and then attempt answers.

All of the models assume whole mantle convection and a primordial state which was at the mantle solidus (necessarily superliquidus for the core) because of accretional and core formation heating. Heat production, rheological parameters, and core phase diagram are chosen so as to reproduce the present observed values of heat outflow, upper mantle temperature and viscosity, and inner core radius for the Earth. Aside from the differences which are necessitated by changes in mass and radius, the primary differences between Earth and other terrestrial planets allowed for in the models are variations in core composition and allowance for a "rigid" lithosphere (for one-plate planets). A few models considered possible changes between planets in heat production and rheological parameters.

We consider the following potential criticisms and questions: (1) To what extent can parameterized mantle convection be expected to provide quantitative estimates of mantle and core cooling? (2) What constraints exist on the compositions of the cores? What is the relationship between core composition and magnetic field? (3) How sensitive are the model results to uncertainties in the numerous input parameters and their variations between planets, including initial conditions, radiogenic heat sources, core and mantle adiabats, mantle rheology, and composition? (4) What implications do the models have for the thermal and magnetic histories of the terrestrial planets? Are there any testable predictions?

We begin by considering the application of parameterized convection. Although this

method of treating subsolidus convection has been applied with some success to the thermal histories of the Earth and other planets (Sharpe and Peltier, 1978, 1979; Stevenson and Turner, 1979; Cassen et al., 1979; Schubert et al., 1979a,b, 1980; Davies, 1980; Cook and Turcotte, 1981; Richter and McKenzie, 1981; Schubert and Spohn, 1981; Spohn and Schubert, 1982), the proper method of parameterization is still debated (Daly, 1980; Nataf and Richter, 1982). The basis of the method is the asymptotic dependence of convective heat transport on Rayleigh number in vigorously convecting systems as typified, for example, in the boundary layer theory of Turcotte and Oxburgh (1967); see also Olson (1981). These asymptotic dependences are strictly valid only for convection of a constant viscosity fluid. However, Schubert et al. (1979b, 1980) have summarized a number of arguments in support of its application to mantle convection. Clearly, different choices of parameters such as  $\beta$ ,  $Ra_{cr}$ ,  $\nu_0$ , and A can lead to different cooling rates and core heat fluxes. However, previous work has shown that the choices are strongly constrained by the requirements that the correct present-day upper mantle temperature and viscosity be obtained for the Earth. The convection parameterization does not provide an unambiguous formulation for the treatment of boundary layers, and we have considered both global [Eq. (18)] and local [Eq. (20)] criteria for the thickness of the lower boundary layer in Earth models. Models E1 and E2 allow this lower boundary layer to be thinned by imposing a local stability criterion. These models cool the core more efficiently and predict higher present-day core heat fluxes than E3 and E4, in which the boundary layer thickness is globally determined. Nevertheless, models E3 and E4 are equally successful in reproducing the correct present-day inner core size, mantle viscosity, and surface heat flux. We have applied the local boundary layer criterion in all our models of planets other than the

Earth, but our results for the Earth indicate that our general conclusions would not be different if we had used a global criterion.

The parameterization of the core liquidus in our models was based on sulfur as the light alloying constituent. It is important to realize, however, that the models would be *identical* if we had chosen  $\alpha_c$  differently in Eq. (3), and compensated by adjusting  $x_0$ (the initial abundance of light constituent). For example, a model in which  $\alpha_c = 2$ ,  $x_0 =$ 0.1 would have *identical* behavior to a model in which  $\alpha_c = 4$ ,  $x_0 = 0.05$  or  $\alpha_c = 1$ ,  $x_0 = 0.2$ , provided all other parameters are unaltered. It is also important to realize that the validity of our models is not contingent on precise knowledge of the melting curve of pure iron, because the models are adjusted to obtain the correct inner core size for the present Earth. The timing of inner core nucleation, for example, depends on the ability of the mantle to eliminate core heat and not on the details of the parameterization of the core liquidus.

There are large uncertainties in the amounts and identities of light constituents in the cores of terrestrial planets. Oxygen may be the major light alloving constituent in the cores of Venus and Earth (Ringwood, 1977). It has been argued that incorporation of oxygen may be difficult even if it is thermodynamically preferred (Stevenson, 1981) but McCammon et al. (1983) have proposed a strong eutectic in FeO at high pressures and a core formation model in which large amounts of oxygen enter the core. Sulfur remains a strong candidate because it is cosmochemically available and only 9-12% by weight is required to explain the Earth's core density (Ahrens, 1979). The chondritic model of Anders and Morgan (1980) has 9% sulfur in the Earth's core. Other model compositions, as reviewed by the authors of the Basaltic Volcanism Study Project (1981) have 5 to 26% by weight of sulfur. Cosmochemical model compositions of Venus have sulfur concentrations varying from 0 to 10% (Basaltic Volcanism Study Project, 1981). The chondritic model of Anders and Morgan (1980) has 5.5 wt% sulfur. It is often argued that Venus may have less sulfur than Earth because it formed closer to the Sun. The model of Ringwood (1977) has 4.9 wt% sulfur and 9.8 wt% oxygen, a composition that would probably keep Venus from freezing an inner core. The models of Jagoutz *et al.* (1979) and Palme *et al.* (1978) have 1% sulfur and 8% oxygen. It is much more likely that Venus' core is completely fluid than almost completely frozen, given that these are the two choices of models consistent with the absence of a substantial magnetic field.

The composition of the Martian core is similarly uncertain. It is conceivable that Mars' core contains even more than 15 wt% sulfur (Basaltic Volcanism Study Project, 1981) but Morgan and Anders (1979) have modeled Mars with an iron-rich core containing only 3.5 wt% sulfur. The pressures are too low for significant incorporation of oxygen into the core. The amount of sulfur in the Mercurian core is not well constrained by cosmochemistry. A strict interpretation of equilibrium condensation scenarios (Lewis, 1972) would lead to negligible amounts, but a significant amount of radial mixing of planetesimals within the primordial solar nebula must cause Mercury to accrete bodies that include more volatile constituents.

An important aspect of our models is that as the core freezes, the lighter constituent is concentrated into the remaining outer fluid shell and the liquidus is lowered, thereby retarding inner core growth. This is the reason why none of our models achieved complete freezing. The self-regulated present-day mantle temperature in all of the terrestrial planets is higher than the FeS eutectic temperature at the core-mantle boundary. Young and Schubert (1974) obtained lower temperatures and a completely solidifed Martian core in their finite amplitude, constant-viscosity convection models, even allowing for the strongly depressed FeS eutectic temperature. The main differences between their model and those presented here is that the strongly temperature-dependent rheology of our models prevents cooling to the FeS eutectic at the present day. Of course, the rheology of the Martian mantle is not well known and we cannot exclude the possibility of complete freezing, as found by Young and Schubert (1974). However, it does require a "soft" rheology (one for which a viscosity of only  $10^{16}$  m<sup>2</sup> sec<sup>-1</sup> is obtained at a temperature as low as 0.6 to 0.65 of the mantle solidus).

It is clear from these considerations that core composition is too poorly known to enable a clear choice of present core state on the basis of our models. However, it is also clear that each core is likely to contain significant alloying constituents and that complete or nearly complete core freezing is unlikely.

We turn now to a consideration of other unknown parameters and their possible effect on our conclusions. We have assumed that the mantles of the terrestrial planets all have the same thermal and material properties as the Earth (Table I). We have also relied on rather crude estimates of core densities and pressures for Venus, Mars, and Mercury, derived from a simple twolayer model. However, they are within the range of estimates by others (Siegfried and Solomon, 1974; Johnston and Toksöz, 1977; Ringwood and Anderson, 1977). In the particularly important case of Venus, a detailed calculation with realistic equations of state yielded 286 GPa for the central pressure, compared with 290 GPa for the simple two-layer model, indicating that our estimates have adequate accuracy. A more serious problem arises in estimates of core adiabats, mantle rheological parameters, and radiogenic heat production. Our Grüneisen parameter  $\gamma$  for the core was based on Stacey (1977a), for which  $\gamma \propto \rho^{-0.6}$ . However, we did consider models in which  $\gamma \propto \rho^{-1}$  (E6) and  $\gamma = 1.7$  (E5). These models provide equally satisfactory descriptions of the present state of the Earth as our nominal model (E1). We conclude that changes in  $\gamma$  lead to only minor changes in inner core freezeout timing and core heat flux provided, of course, that the model is adjusted to ensure the correct present-day inner core size. The value of  $\gamma$  is not likely to be sensitive to core chemistry (Stevenson, 1981) and choices which yield satisfactory Earth models should be applicable to the other planets. We have not varied rheological parameters, except in the single case of the Venus model V3 where A was reduced from 5.2  $\times$  10<sup>4</sup> to 4.8  $\times$  10<sup>4</sup> °K<sup>-1</sup>. As expected, a reduction in A causes lower mantle temperatures and earlier freezeout of an inner core. A reduction in radiogenic heating also causes lower mantle temperatures and earlier freezeout of an inner core. There is currently very little understanding of how mantle rheology might vary from planet to planet and we cannot assess whether A is likely to be larger or smaller in Venus, say, than Earth. Radiogenic heating might be less in the Venus mantle than Earth's mantle if Venus has less <sup>40</sup>K or a very thick basaltic crust. Radiogenic heating might be higher in Mars if it retained more of the chondritic abundance of potassium. None of these possibilities can be assessed with confidence at present and the resulting uncertainties must be acknowledged in considering the consequences of the models.

All of the models begin with the mantle at the solidus. This is a reasonable assumption for Earth and Venus, where the combined effects of accretion and core formation are more than capable of achieving this temperature (Kaula, 1980; Shaw, 1979). It is also valid for Mars and Mercury provided more than about 10% of the accretional energy is retained as internal heat. The subsequent evolution of all the planets is rather insensitive to this initial condition, provided it is hot enough that the heat output greatly exceeds radiogenic heat production, because convective self-regulation rapidly cools the planet to a state that approaches (but never reaches) equilibrium with the radiogenic heat production (Schubert et al., 1980). We cannot exclude the possibility of primordial inner cores, however. This would depend on details of the core formation dynamics.

We turn finally to the implications of our models for each of the planets considered. A very interesting feature of our Earth models is the nucleation of an inner core late in its thermal history. Since the Earth's magnetic field is at least 3.5 by old (McElhinny and Senanayake, 1980), the mode of powering the dynamo may have changed during Earth history. In the Earth's early thermal history, the magnetic field was probably powered by the heat engine of thermal convection, the heat being obtained from secular cooling of a fluid core. This has the associated low efficiency inherent in a heat engine (Gubbins, 1977a). After inner core growth was initiated, 1.5 to 2.5 by ago, the release of gravitational energy rapidly became the dominant energy source for the dynamo. Latent heat release may also be important but has diminished effectiveness because of the Carnot efficiency factor associated with any purely thermal energy source. The outer cores of models E1 and E2 cool approximately 70°K from the beginning of inner core freezing to the present; equivalent to average energy releases from secular cooling alone of 5  $\times$  $10^{11}$  W (E1) and 4 ×  $10^{11}$  W (E2). The total gravitational and latent heat released during this time is much larger and corresponds to average powers of 2  $\times$  10<sup>12</sup> W (E1) and 3  $\times$ 10<sup>12</sup> W (E2). Gubbins et al. (1979) have made a detailed estimate of the energy supply necessary to drive the dynamo. They estimate that  $2.5 \times 10^{12}$  W is needed to maintain a 10- to 20-mT toroidal field in a magnetic configuration corresponding to the Kumar and Roberts (1975) dynamo. This is compatible with our models. If this dynamo were maintained by cooling of a completely fluid core, then  $8 \times 10^{12}$  W would be required, clearly incompatible with any reasonable present day core heat flux.

Estimates of this kind are uncertain be-

cause there is no consensus concerning the magnitude of the Earth's toroidal field. Indirect estimates suggest that the toroidal field may be 10 mT or more (Hide and Roberts, 1979), probably requiring a gravitational energy source. It is of interest to evaluate a nominal magnetic history of the Earth by equating the energy available for dynamo generation, as given by our models, to the ohmic dissipation:

$$\Phi = E_{\rm G} \frac{dm}{dt} + \eta \left( L \frac{dm}{dt} - \frac{dE_{\rm th}}{dt} - 4\pi R_{\rm c}^2 F_{\rm cond} \right), \quad (27)$$

J ....

where  $\Phi$  is the ohmic dissipation,  $\eta$  is a Carnot efficiency factor,  $dE_{th}/dt$  is the rate of change of heat content of the core, and  $F_{cond}$  is the heat flux conducted along the adiabat. If latent heat release is unimportant, then  $\eta \simeq 0.06$ . If latent heat release dominates, then  $\eta \simeq 0.2$ . We define a nominal nondimensional field strength H(t) in units of the present-day field strength by

$$H(t) = [\Phi(t)/\Phi(4.5 \text{ by})]^{1/2},$$
 (28)

since ohmic dissipation scales as the square of the current or field. This nominal field, shown in Fig. 9, should not be interpreted as the observed (dipole) field since it is possible that the toroidal field can change without a similar change in the poloidal field and vice versa. Nevertheless, it provides a crude measure of possible secular field changes on billion-year timescales. The most striking feature is the low nominal field strength in the period preceding nucleation of the inner core. Existing paleomagnetic evidence does not provide support for this possibility (McElhinny and Evans, 1968) but paleofield determinations are uncertain. The abrupt change at onset of inner core growth might conceivably show up in other aspects of the geomagnetic field such as polarity reversal rate. This may be more amenable to observational test.

Our Venus models admit present core



FIG. 9. Nominal field strength through geologic time as defined by Eqs. (27) and (28). One solid curve is for E1 (energy release per unit inner core mass of  $10^6$  J kg<sup>-1</sup>) and one dashed curve corresponds to E2 ( $2 \times 10^6$  J kg<sup>-1</sup>). The other solid and dashed curves are for Venus model V1 but with the field normalized according to the energy releases of E1 and E2, respectively.

states similar to the Earth, with an inner core and a convective outer core, but they also admit completely fluid, stably stratified cores and cores which are mostly frozen. Completely fluid models are only marginally possible for Earth-like parameters and arise because Venus has a somewhat lower central pressure (about 290 GPa compared with 360 GPa for the Earth) but somewhat higher mantle temperatures. If Venus is a one-plate planet (Phillips et al., 1981), then temperatures may be slightly higher still, further retarding onset of inner core growth. Since models with almost frozen cores require implausibly low amounts of light alloying constituents, we favor a completely fluid core as an explanation for the absence of a substantial magnetic field for Venus. This has two interesting implications. One implication is that Venus once had a substantial magnetic field (see Fig. 9) which died  $\sim 1.5$  by ago. It follows, for example, that calculations involving solar wind influences on a primordial Venus atmosphere could be incorrect if they assume that the present magnetic state has persisted throughout geologic time. Another implication is that Venus will eventually nucleate an inner core and this might cause revival of the dynamo. The energy release from freezeout would first need to overcome the subadiabatic outer core state that has developed in the meantime, so the reintroduction of a field might take several billion years if it happened at all.

Our Mars models admit present core states similar to the Earth, with an inner core and a convective outer core, but they also admit completely fluid cores and one can also imagine models in which the core might be close to complete freezing. We favor a completely fluid core since it is predicted for a cosmochemically plausible sulfur content of 15% or more by weight and provides an explanation for the absence of a substantial magnetic field. On Mars, unlike Venus, there is some prospect of eventually testing our models by measuring the natural remanence of rock samples of age greater than 3.5 by bp. (Venus is impractical in this regard because the surface temperature exceeds the blocking temperature of likely magnetic minerals.)

Our Mercury models predict a large solid inner core but the persistence of a significant fluid FeS layer to the present day. The depth of this layer is roughly  $10^4 x_0$  km, where  $x_0 \ll 1$  is the initial sulfur mass fraction in the core. A thin-shell dynamo is the favored interpretation of Mercury's magnetic field. It is possible that the existence of a fluid shell can be detected by measurements of the rotational state made by a Mercury orbiter (Peale, 1981).

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