EFFECT OF SUBSURFACE DIFFERENTIAL ROTATION (IMPOSED PROFILE OF THE DIFFERENTIAL ROTATION)

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1. Equations

1.1. Self-consistent Boussinesq formulation.

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1.1a}$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla \pi + \nabla^2 \boldsymbol{u} + \Theta \boldsymbol{r} + \tau \boldsymbol{u} \times \boldsymbol{k} + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B}, \tag{1.1b}$$

$$P(\partial_t \Theta + \boldsymbol{u} \cdot \nabla \Theta) = \nabla^2 \Theta + R \boldsymbol{r} \cdot \boldsymbol{u}, \tag{1.1c}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.1d}$$

$$\partial_t \boldsymbol{B} = P_m^{-1} \nabla^2 \boldsymbol{B} + \nabla \times (\boldsymbol{u} \times \boldsymbol{B}). \tag{1.1e}$$

The Rayleigh number R, the Coriolis number τ, the Prandtl number P and the magnetic Prandtl number P_m are defined by

$$R = \frac{\alpha \gamma \beta d^6}{\nu \kappa}, \quad \tau = \frac{2\Omega d^2}{\nu}, \quad P = \frac{\nu}{\kappa}, \quad P_m = \frac{\nu}{\lambda}, \tag{1.2}$$

1.2. Imposed differential rotation.

Observations of solar differential rotation show a profile decreasing with radius at the top 15% of the convective zone. Busse [1] finds that a subsurface convection layer may generate mean flows that will act to reduce the amplitude of differential rotation.

The subsurface convection layer cannot be resolved in our global model. Thus, in order to capture this effect we impose an additional differential rotation flow profile to our self-consistent dynamo model.

Denote by $\boldsymbol{U}_I$ the imposed additional profile of the differential rotation with components,

$$\boldsymbol{U}_I = \Big(0, 0, \mathrm{tr}(r) \sin\theta\Big),$$

where

$$\mathrm{tr}(x) = \begin{cases} \beta \dfrac{x-b}{1-b}, & b < x \\ 0, & b > x, \end{cases}$$

with $\beta < 0$ and $b \in (0,1)$, and $x \in (0,1]$;

Let us think of $\boldsymbol{U}_I$ as being the background reference state, generated by a force associated with a extra thin surface layer of convection that cannot be otherwise resolved. The equations for the deviations from that basic state are,

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1.3a}$$

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} + \boldsymbol{U}_I) \cdot \nabla \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{U}_I = -\nabla \pi + \nabla^2 \boldsymbol{u} + \Theta \boldsymbol{r} + \tau (\boldsymbol{u} + U_I) \times \boldsymbol{k} + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B}, \tag{1.3b}$$

$$P\Big(\partial_t \Theta + (\boldsymbol{u} + \boldsymbol{U}_I) \cdot \nabla \Theta\Big) = \nabla^2 \Theta + R \boldsymbol{r} \cdot \boldsymbol{u}, \tag{1.3c}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.3d}$$

$$\partial_t \boldsymbol{B} = P_m^{-1} \nabla^2 \boldsymbol{B} + \nabla \times \Big((\boldsymbol{u} + \boldsymbol{U}_I) \times \boldsymbol{B}\Big). \tag{1.3e}$$

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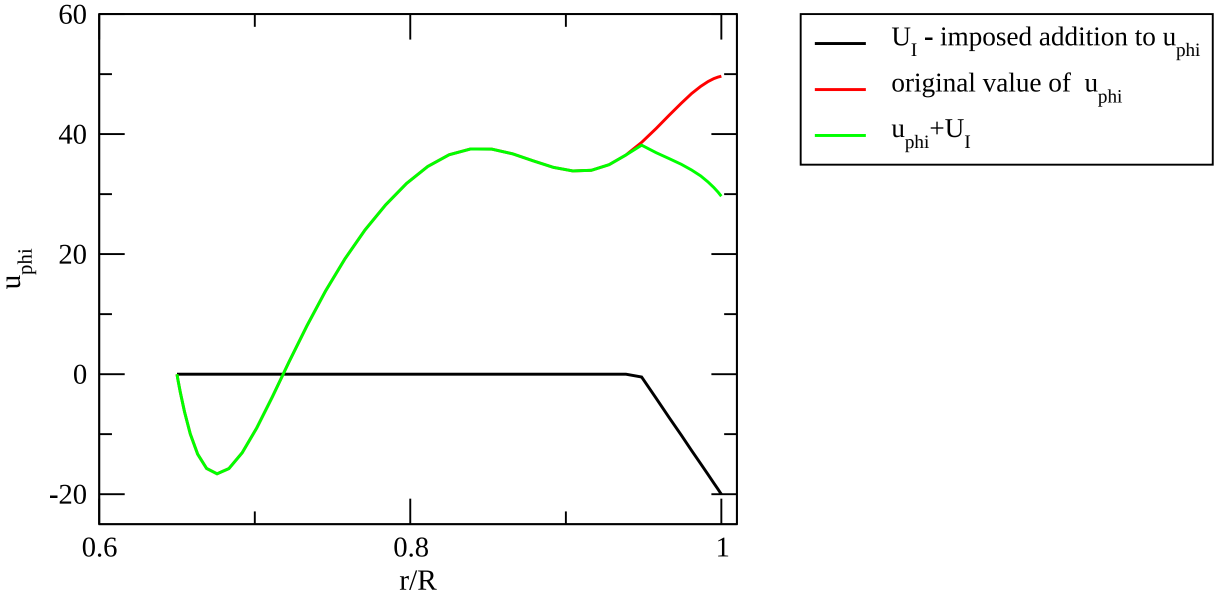

FIGURE 1.

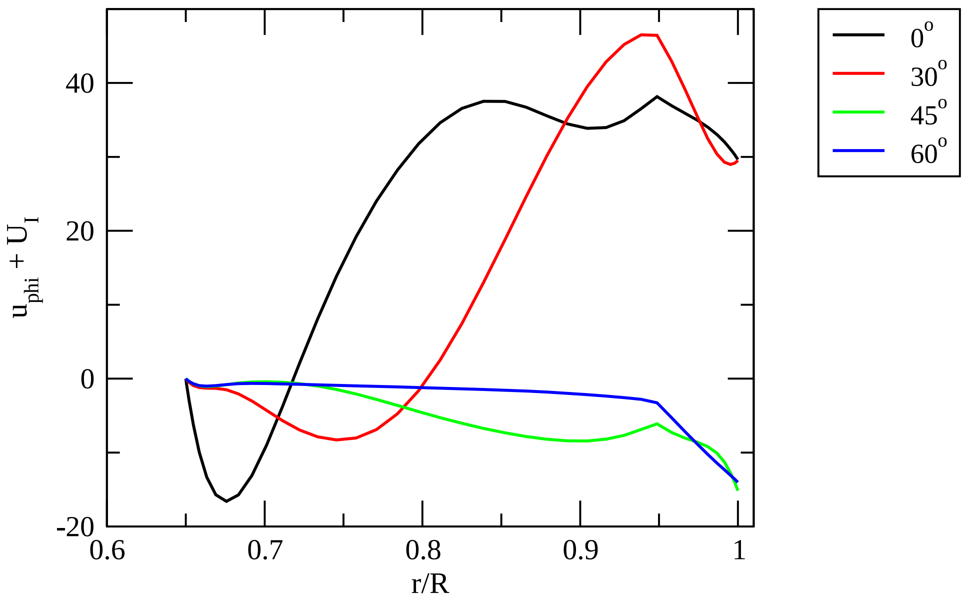

FIGURE 2.

2. Initial try

An initial test is to add the imposed flow only to the induction term $\nabla \times (\boldsymbol{u} + \boldsymbol{B})$, given in red. This should lead to drift of toroidal field lines towards the EQUATOR without affecting convection. We will use that simple model to adjust the parameters (slope and magnitude) of the imposed profile in an attempt to achive the desired effect.

The next step will be to take into account the nonlinear effects of that flow on convection as given by the terms in blue.

Unfortunately, an initial simulation DOES NOT show a very pronounced effect.
Numerical simulation of a test case $\eta = 0.65$ $P = 1$, $\tau = 2000$, $R = 120000$, $P_m = 5$.

- Imposed flow U_I: e065p1t2r120000m1p5mvbcFD Click for MOVIE
- Imposed flow U_I: e065p1t2r120000m1p5mvbcFDb100 Click for MOVIE
- NO imposed flow.

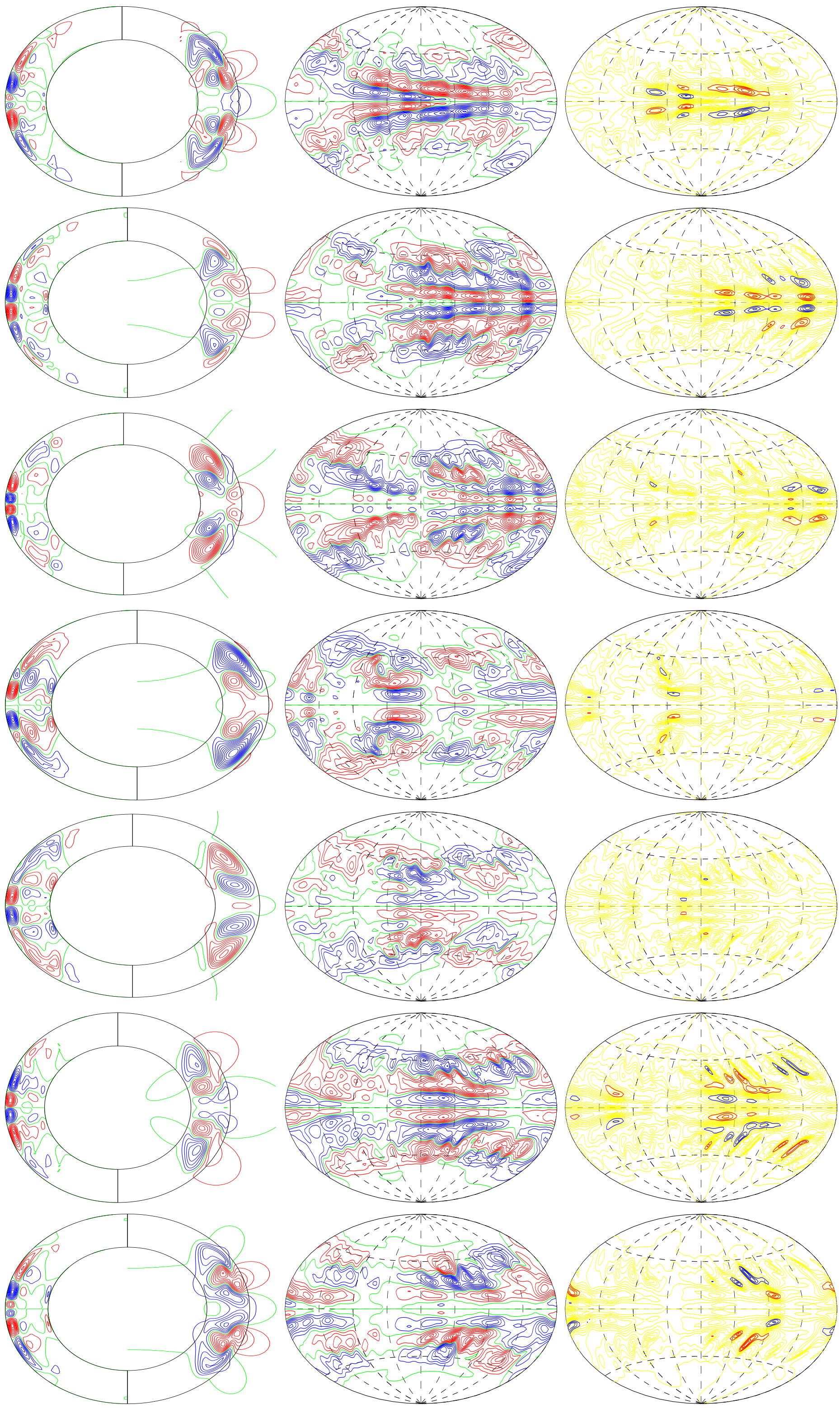

FIGURE 3. A period of oscillation in the case $\eta = 0.65$, $P = 1$, $\tau = 2000$, $R_e = 120000$, $P_m = 5$, stress-free velocity boundary condition at $r = r_o$ and no-slip at $r = r_i$. IMPOSED DIFF ROT.

Click for MOVIE

Nice oscillation

References

1. F. H. Busse, *Convection in the presence of an inclined axis of rotation with applications to the sun*, Solar Physics **245** (2007), 27–36.