# Minimal models of the Solar cycle

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**Abstract.** We discuss the extent to which minimal self-consistent models of convection-driven dynamos may reproduce the Solar cycle.

PACS numbers: ?? ?? ?? (specify here)

#### 1. Figures dated - Nov 2011

The following figures are available.

- Please, let me know which ones you would like to use.
- Please, let me know of new figures requited, and I'll find time to prepare them.



Figure 1: Convection-driven dynamos as a function of R, P and  $P_m$  for  $\tau = 2000$ . Decaying dynamos are indicated by black dots, MD dynamos are indicated by blue diamonds, FD dynamos are indicated by red stars, quadrupolar dynamos are indicated by green triangles, and coexisting MD and FD dynamos are indicated by pink squares. Full symbols correspond to  $\eta = 0.65$ , and empty symbols correspond to  $\eta = 0.6$ .



Figure 2: Co-existing nonlinear dynamo solutions at identical parameter values in the case  $\eta = 0.65 \tau = 2000$ , R = 150000 and  $P/P_m = 0.2$ . The fluctuating poloidal magnetic energy component  $\widetilde{M}_p$  (solid circles) and the mean poloidal magnetic energy component  $\overline{M}_p$  (empty circles) are scaled on the left ordinate, while the Nusselt number at  $r = r_i$  (solid diamonds) is scaled on the right ordinate. Cases started from initial conditions on the FD branch are shown in red, and those started from initial conditions on the MD branch are shown in blue.

TO DO:

- 1) Put Nui in a separate panel below the main one.
- 2) Fine-tune sizes, labels, line thickness etc.



Figure 3: Half a period of oscillation in the case  $\eta = 0.65$ , P = 0.8,  $\tau = 1750$ ,  $R_e = 100000$ ,  $P_m = 4$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . Lines of constant g at r = 0.9, of  $B_{horiz}$  at r = 0.9, and  $B_r$  at r = 1 are shown in the first, second and last column, respectively. The third column shows meridional lines of constant  $\overline{B}_{\varphi}$  to the left and of  $r \sin \theta \partial_{\theta} \overline{h}$  to the right. The time interval between the snapshots is 0.0168.

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Figure 4: Profiles of the differential rotation  $\overline{u}_{\varphi}$  as a function of  $\theta$  at  $r = r_o$  in the case  $\eta = 0.65$ , P = 0.8,  $\tau = 1750$ ,  $R_e = 100000$ ,  $P_m = 4$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . e065p08t1.75r100000m1p4mvbcFD.per



Figure 5: Contour plots of the  $B_{\varphi}|_{m=0,1}$  at r = 0.96 (top) and  $B_r|_{m=0,1}$  at r = 1 (bottom) as a function of time (x axis) and latitude  $\theta$  (y axis) in the case  $\eta = 0.65$ , P = 0.8,  $\tau = 1750$ , R = 100000,  $P_m = 4$ ,  $\beta = 0$  with mixed velocity boundary conditions. e065p08t1.75r100000m1p4mvbcFD.perTO DO: 1) Re-plot in color

- 2) Remove spurious lines near ends of plot
- 3) Fine-tune sizes, labels, line thickness etc.

Minimal models of the Solar cycle



Figure 6: Half a period of oscillation in the case  $\eta = 0.65$ , P = 1.2,  $\tau = 2000$ ,  $R_e = 120000$ ,  $P_m = 4.5$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . The first column shows meridional lines of constant  $\overline{B}_{\varphi}$  to the left and of  $r \sin \theta \partial_{\theta} \overline{h}$  to the right. Lines of constant  $B_{horiz}$  at r = 0.9, of g at r = 0.9, and  $B_r$  at r = 1 are shown in the second, third and fourth column, respectively. The time interval between the snapshots is 0.0224.

e065 p1.2 t2 r120000 m1 p4.5 mv bc FD



Figure 7: Profiles of the differential rotation  $\overline{u}_{\varphi}$  as a function of  $\theta$  at  $r = r_o$  in the case  $\eta = 0.65$ , P = 1.2,  $\tau = 2000$ ,  $R_e = 120000$ ,  $P_m = 4.5$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . e065p1.2t2r120000m1p4.5mvbcFD



Figure 8: Contour plots of the  $B_{\varphi}|_{m=0,1}$  at r = 0.9 (top) and  $B_r|_{m=0,1}$  at r = 1 (bottom) as a function of time (x axis) and latitude  $\theta$  (y axis) in the case  $\eta = 0.65$ , P = 1.2,  $\tau = 2000$ , R = 120000,  $P_m = 4.5$ ,  $\beta = 0$  with mixed velocity boundary conditions. e065p1.2t2r120000m1p4.5mvbcFD TO DO:

- 1) Re-plot in color
- 2) Remove spurious lines near ends of plot
- 3)Fine-tune sizes, labels, line thickness etc.



Figure 9: A period of oscillation in the case  $\eta = 0.65$ , P = 1,  $\tau = 2000$ ,  $R_e = 100000$ ,  $P_m = 4.5$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . The first row shows contour lines of  $B_{horiz}$  at r = 0.9 and the second row shows contour lines of g at r = 0.9. The time interval between the snapshots is 0.0224. e065p1t2r100000m1p4.5mvbcB.per02 TO DO: 1) Re-plot in column format



Figure 10: A period of oscillation in the case  $\eta = 0.65$ , P = 1,  $\tau = 2000$ ,  $R_e = 100000$ ,  $P_m = 4.5$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . The first row shows meridional lines of constant  $\overline{B}_{\varphi}$  to the left and of  $r \sin \theta \partial_{\theta} \overline{h}$  to the right and the second row shows contour lines of  $B_r$  at r = 1. The time interval between the snapshots is 0.0224.

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Figure 11: A period of oscillation in the case  $\eta = 0.65$ , P = 1,  $\tau = 2000$ ,  $R_e = 100000$ ,  $P_m = 4$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . The first row shows contour lines of  $B_r$  at r = 1 and the second row shows contour lines of  $-\frac{\partial g}{\partial \theta}$  at r = 0.9. The time interval between the snapshots is 0.0308. e065p1t2r100000m1p4mvbcFD TO DO: 1) Re-plot in column format



Figure 12: A period of oscillation in the case  $\eta = 0.65$ , P = 1,  $\tau = 2000$ ,  $R_e = 100000$ ,  $P_m = 4$ ,  $\beta = 0$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . The first row shows contour lines of  $B_{horiz}$  at r = 0.9 and the second row shows contour lines of g at r = 0.9. The time interval between the snapshots is 0.0308. e065p1t2r100000m1p4mvbcFD TO DO: 1) Re-plot in column format



Figure 13: A period of oscillation in the same case and at the same instances as in the previous figure. The first row shows meridional lines of constant  $\overline{B}_{\varphi}$  to the left and of  $r \sin \theta \partial_{\theta} \overline{h}$  to the right and the second row shows contour lines of  $B_r$  at r = 1. The time interval between the snapshots is 0.0308. e065p1t2r100000m1p4mvbcFD TO DO: 1) Re-plot in column format



Figure 14: Contour plots of the mean  $\overline{u}_{\varphi} - \langle \overline{u}_{\varphi} \rangle_{\text{time}}$  as a function of time (x axis) and latitude  $\theta$  (y axis) in the case  $\eta = 0.65$ , P = 1,  $\tau = 2000$ , R = 100000,  $P_m = 4$ ,  $\beta = 0$  with mixed velocity boundary conditions.

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1) Re-plot in color

2) Remove spurious lines near ends of plot

3)Fine-tune - sizes, labels, line thickness etc.



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Figure 15: Contour plots of the  $B_{\varphi}|_{m=0,2}$  at r = 0.9 (top) and  $B_r|_{m=0,2}$  at r = 1 (bottom) as a function of time (x axis) and latitude  $\theta$  (y axis) in the case  $\eta = 0.65$ ,  $P = 1, \tau = 2000, R = 100000, P_m = 4, \beta = 0$  with mixed velocity boundary conditions. e065p1t2r100000m1p4mvbcFD TO DO:

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2) Remove text in figure, color legents etc 3)Fine-tune - sizes, labels, line thickness etc.



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Figure 16: Contour plots of the  $B_{\varphi}|_{m=0,1}$  at r = 0.96 (top) and  $B_r|_{m=0,1}$  at r = 1 (bottom) as a function of time (x axis) and latitude  $\theta$  (y axis) in the case  $\eta = 0.65$ ,  $P = 1, \tau = 2000, R = 100000, P_m = 4, \beta = 0$  with mixed velocity boundary conditions. e065p1t2r100000m1p4mvbcFD TO DO: 1) Re-plot in color 2) Remove spurious lines near ends of plot 3)Fine-tune - sizes, labels, line thickness etc.



Figure 17: Nearly a full period of oscillation in the case  $\eta = 0.65$ , P = 1,  $\tau = 2000$ ,  $R_e = 140000$ ,  $P_m = 3.5$ ,  $\beta = 1$  with stress-free velocity boundary condition at  $r = r_o$  and no-slip at  $r = r_i$ . The left column shows contour lines of  $B_r$  at r = 1 and the right column shows contour lines of  $-\frac{\partial g}{\partial \theta}$  at r = 0.9. The time interval between the snapshots is 0.0224.

e065 p1t2r140000 m1 p3.5 mvbcFDsl1.per

#### 2. Figures dated - 03 Jan 2011



Figure 35: (color online) A dynamo oscillation in the case  $\tau = 2000$ , R = 150000, P = 1,  $P_m = 4.5$ ,  $\beta = 0$ , stress-free at  $r_o$  and no-slip at R - i. The first column shows meridional lines of constant  $\overline{B_{\varphi}}$  on the left and poloidal field lines,  $r \sin \theta \partial \overline{h} / \partial \theta$  on the right. The second column shows lines of constant  $\partial g / \partial \theta$  at r = 0.9 corresponding to -0.9, -0.8, -0.7, 0.7, 0.8, 0.9 of the maximum absolute value of  $\partial g / \partial \theta$ , and the third column shows lines of constant  $B_r$  at  $r = r_o$ . The last column shows  $\operatorname{Re}(\partial g^{m=2} / \partial \theta)$  on the left and  $\operatorname{Im}(\partial g^{m=2} / \partial \theta)$  on the right. The five rows are separated equidistantly in time by  $\Delta t = 0.028$ .

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Figure 36: (color online) A dynamo oscillation in the case  $\tau = 2000$ , R = 110000, P = 1,  $P_m = 4.5$ ,  $\beta = 0$ , stress-free at  $r_o$  and no-slip at R - i. The first column shows meridional lines of constant  $\overline{B_{\varphi}}$  on the left and poloidal field lines,  $r \sin \theta \partial \overline{h} / \partial \theta$  on the right. The second column shows lines of constant  $\partial g / \partial \theta$  at r = 0.9 corresponding to -0.9, -0.8, -0.7, 0.7, 0.8, 0.9 of the maximum absolute value of  $\partial g / \partial \theta$ , and the third column shows lines of constant  $B_r$  at  $r = r_o$ . The last column shows  $\operatorname{Re}(\partial g^{m=2} / \partial \theta)$  on the left and  $\operatorname{Im}(\partial g^{m=2} / \partial \theta)$  on the right. The five rows are separated equidistantly in time by  $\Delta t = 0.0168$ . e065p1t2r110000m1p4.5mvbcFD



Figure 37: (color online) A dynamo oscillation in the case  $\tau = 2000$ , R = 100000, P = 1,  $P_m = 6$ ,  $\beta = 0$ , stress-free at  $r_o$  and no-slip at R - i. The first column shows meridional lines of constant  $\overline{B_{\varphi}}$  on the left and poloidal field lines,  $r \sin \theta \partial \overline{h}/\partial \theta$  on the right. The second column shows lines of constant  $\partial g/\partial \theta$  at r = 0.9 corresponding to -0.9, -0.8, -0.7, 0.7, 0.8, 0.9 of the maximum absolute value of  $\partial g/\partial \theta$ , and the third column shows lines of constant  $B_r$  at  $r = r_o$ . The last column shows  $\operatorname{Re}(\partial g^{m=2}/\partial \theta)$  on the left and  $\operatorname{Im}(\partial g^{m=2}/\partial \theta)$  on the right. The five rows are separated equidistantly in time by  $\Delta t = 0.028$ .