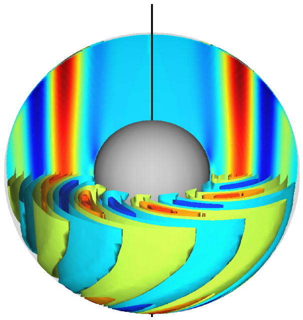

Convection-driven spherical dynamos: bistability and attempts to model the Solar cycle

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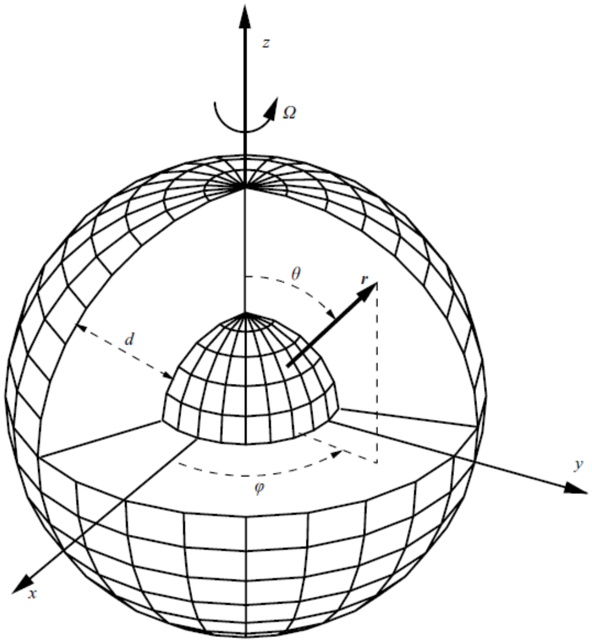
Convective spherical shell dynamos



Model equations & parameters

Boussinesq approximation

$$\nabla \cdot \boldsymbol{u} = 0, \quad \nabla \cdot \boldsymbol{B} = 0,$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} =$$
$$-\nabla \pi - \tau \boldsymbol{k} \times \boldsymbol{u} + \Theta \boldsymbol{r} + \nabla^2 \boldsymbol{u} + \boldsymbol{B} \cdot \nabla \boldsymbol{B},$$

$$P\,(\partial_t \Theta + \boldsymbol{u} \cdot \nabla \Theta) = R\,\boldsymbol{r} \cdot \boldsymbol{u} + \nabla^2 \Theta,$$

$$P_m(\partial_t \boldsymbol{B} + \boldsymbol{u} \cdot \nabla \boldsymbol{B}) = P_m\,\boldsymbol{B} \cdot \nabla \boldsymbol{u} + \nabla^2 \boldsymbol{B}.$$

$$R = \frac{\alpha \gamma \beta d^6}{\nu \kappa}, \quad \tau = \frac{2 \Omega d^2}{\nu}, \quad P = \frac{\nu}{\kappa}, \quad P_m = \frac{\nu}{\lambda}$$

Simitev & Busse, JFM ,2004

Basic state & scaling

$$T_S = T_0 - \beta d^2 r^2 / 2$$

$$\boldsymbol{g} = -d \gamma \boldsymbol{r}$$

Length scale: d

Time scale: d^2/ν

Temp. scale: $\nu^2/\gamma \alpha d^4$

Magn. flux density: $\nu(\mu \varrho)^{1/2}/d$

Boundary Conditions

$$\boldsymbol{r} \cdot \boldsymbol{u} = \boldsymbol{r} \cdot \nabla \boldsymbol{r} \times \boldsymbol{u}/r^2 = 0,$$

$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} \cdot \boldsymbol{B}_{\text{int}} = \hat{\boldsymbol{e}}_{\boldsymbol{r}} \cdot \boldsymbol{B}_{\text{ext}},$$

$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} \times \boldsymbol{B}_{\text{int}} = \hat{\boldsymbol{e}}_{\boldsymbol{r}} \times \boldsymbol{B}_{\text{ext}},$$

$$\Theta = 0, \text{ at } r = r_i \equiv 2/3 \text{ and } r_o \equiv 5/3$$

Numerical Methods

3D non-linear problem:

Toroidal-poloidal representation

$$\boldsymbol{u} = \nabla \times (\nabla v \times \boldsymbol{r}) + \nabla w \times \boldsymbol{r} \ , \qquad \boldsymbol{B} = \nabla \times (\nabla h \times \boldsymbol{r}) + \nabla g \times \boldsymbol{r}$$

Spectral decomposition in spherical harmonics and Chebyshev polynomials

$$x = \sum_{l,m,n} X^m_{l,n}(t)\, T_n(r)\, P^m_l(\cos\theta) e^{im\varphi} \qquad \text{where } x = (v, w, \Theta, g, h)^T$$

Scalar equations

$$\partial_t X^m_{l,n} = \hat{\mathcal{L}} X^m_{l,n} + N^m_{l,n}(X) \qquad \text{where } \hat{\mathcal{L}} X^m_{l,n}\text{: linear}, N^m_{l,n}(X)\text{: non-linear}$$

Pseudo-spectral method. Time-stepping: Crank-Nicolson & Adams-Bashforth

$$[X^m_{l,n}]^{k+1} = \left(1 - \frac{\Delta t}{2}\hat{\mathcal{L}}\right)^{-1} \left\{ \left(1 + \frac{\Delta t}{2}\hat{\mathcal{L}}\right) [F^m_{l,n}]^k + \frac{\Delta t}{2}\left(3[N^m_{l,n}]^k - [N^m_{l,n}]^{k-1}\right) \right\}$$

Resolution: radial=41, latitudinal=193, azimuthal=96.

Linear problem:

Galerkin spectral method for the linearised equations leading to an eigenvalue problem for the critical parameters.

Tilgner, IJNMF, 1999

Part 1.

Bistability and hysteresis of non-linear dynamos

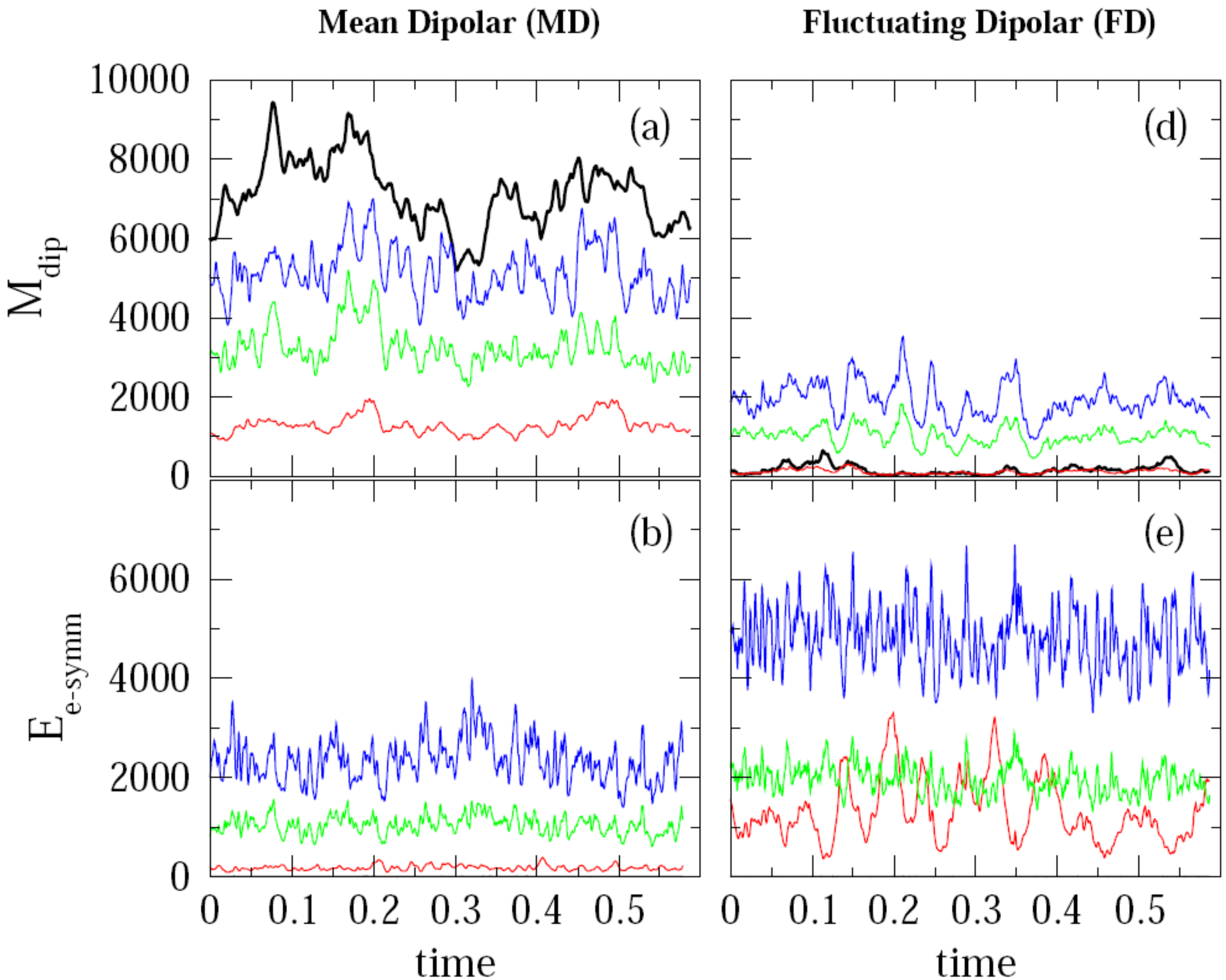
References:

- *Simitev, R., Busse F.H., Bistability and hysteresis of dipolar dynamos generated by turbulent convection in rotating spherical shells, **EPL**, 85, 19001,2009*

- *Simitev, R., Busse F.H., Bistable attractors in a model of convection-driven spherical dynamos, **Physica Scripta** (submitted 1 Dec 2011).*

Two types of dipolar dynamos generated by chaotic convection

Energy densities



$R = 3.5 \cdot 10^6,\ \tau = 3 \cdot 10^4,\ P = 0.75$ and $P_m = 1.5$

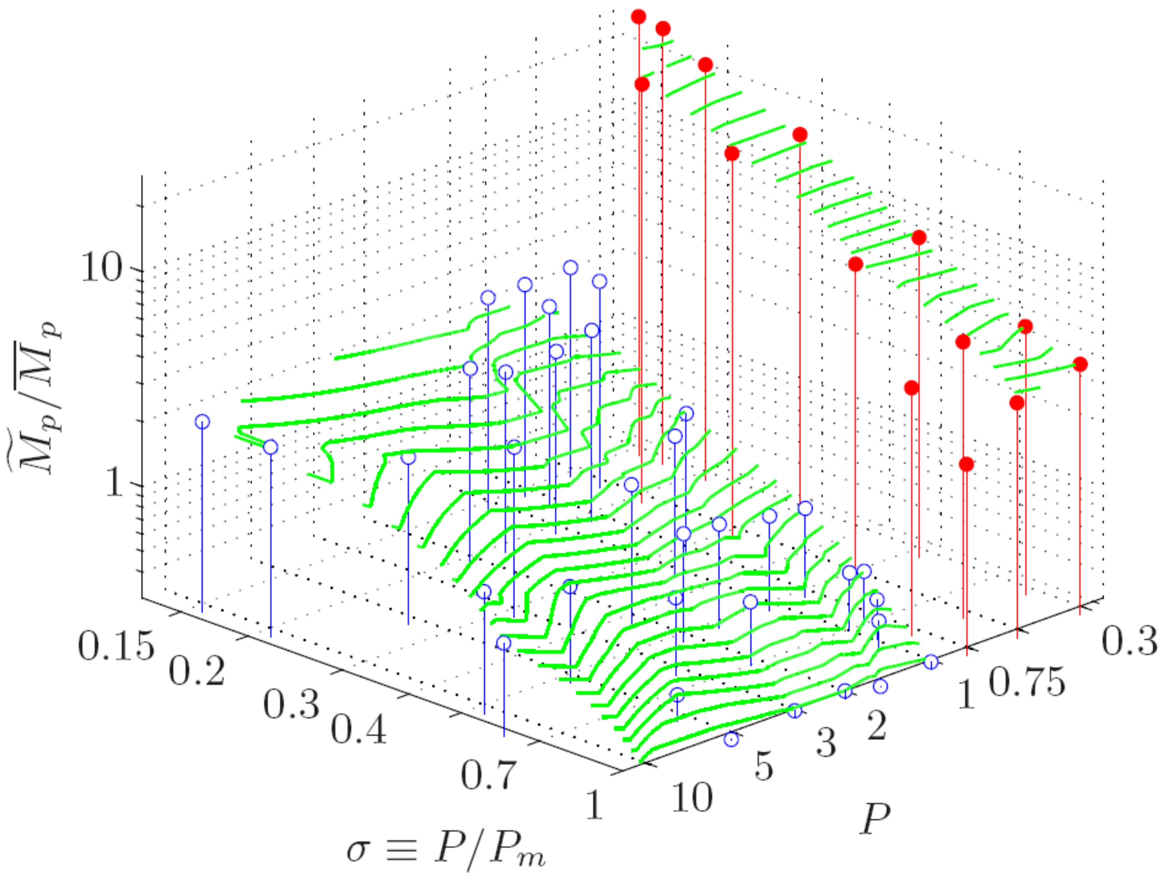
- ***Fully chaotic*** (***large-scale turbulent***) *regime.*
- ***Two chaotic attractors*** *for the same parameter values.*
- *Essential qualitative difference: contribution of the* ***mean poloidal dipolar energy***

	(ab)	(de)
Rm	133.6	196.5
Mdip/Mtot	0.803	0.527

*black.......****mean poloidal***
*green......****fluctuating poloidal***
*red..........****mean toroidal***
*blue.........****fluctuating toroidal***

Regions and transition

Ratio of **fluctuating to mean poloidal magn energy**



$$R = 3.5 \cdot 10^6, \; \tau = 3 \cdot 10^4$$

Two types of dipolar dynamos

- ***Mean Dipolar (MD)***

$$\widetilde{M}_p < \overline{M}_p$$

- ***Fluctuating Dipolar (FD)***

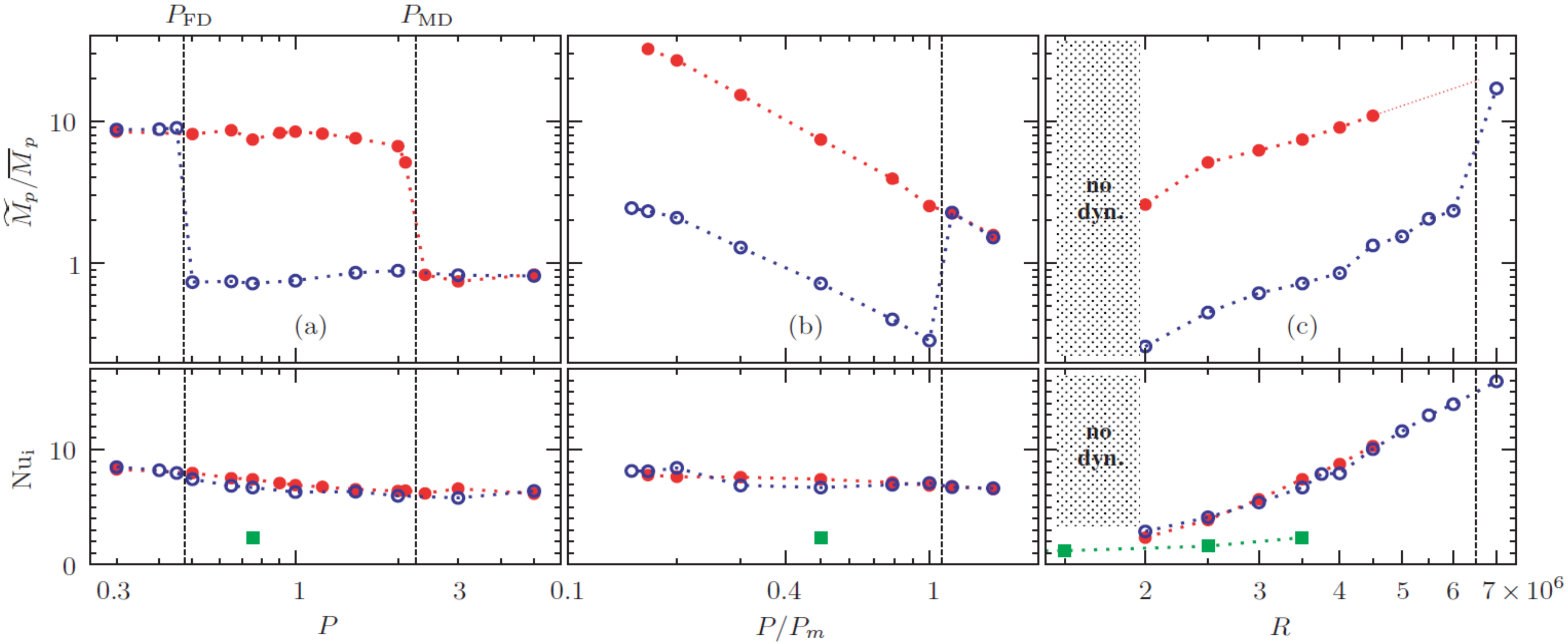
$$\widetilde{M}_p > \overline{M}_p$$

- *MD and FD dynamos correspond to rather **different chaotic attractors** in a fully chaotioc system*
- *The transition between them is not gradual but is an **abrupt jump** as a critical parameter value is surpassed.*
- *The nature of the transition is complicated.*

	MD	FD
Mdip/Mtot	(0.62,1)	(0.41,56)

Bistability and hysteresis in the MD <~> FD transition

***Bistability and hysteresis** in the ratio of fluctuating poloidal to mean poloidal magn energy*



(a) $R = 3.5 \cdot 10^6 \quad P/P_m = 0.5$

(b) $R = 3.5 \cdot 10^6, \; P = 0.75$

(c) $P = 0.75, \; P_m = 1.5$

in all cases: $\tau = 3 \cdot 10^4$

Mean Dipolar (MD)

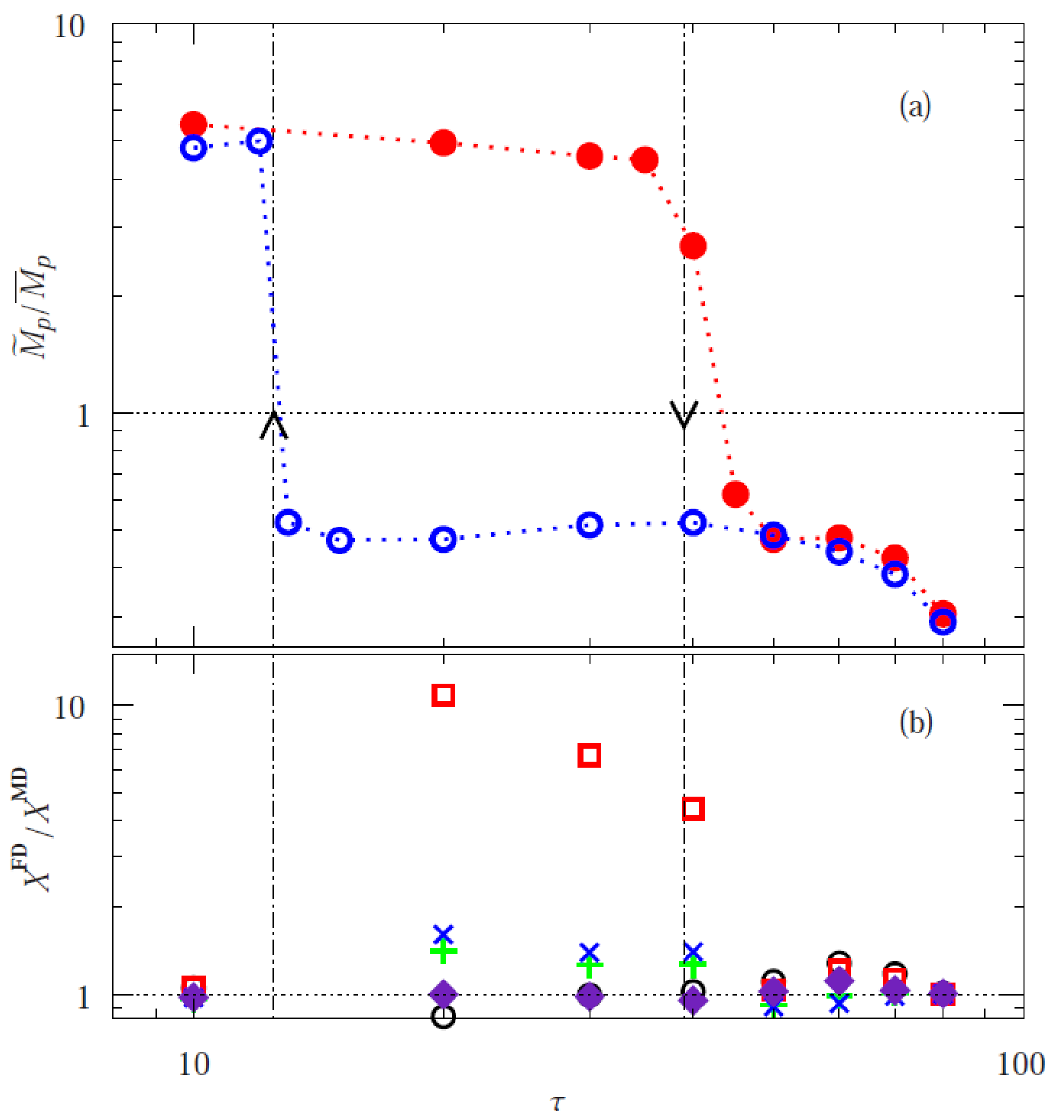
Fluctuating Dipolar (FD)

*The coexistence is **not an isolated phenomenon** but can be traced with variation of the parameters.*

$P_{MD} = 2.2 \qquad P_{FD} = 0.5$

$\sigma_{MD} = 0.07 \qquad \sigma_{FD} = 1$

Bistability and hysteresis as a function of the rotation parameter τ



(a) ratio of fluctuating poloidal to mean poloidal magn energy

Mean Dipolar (MD)

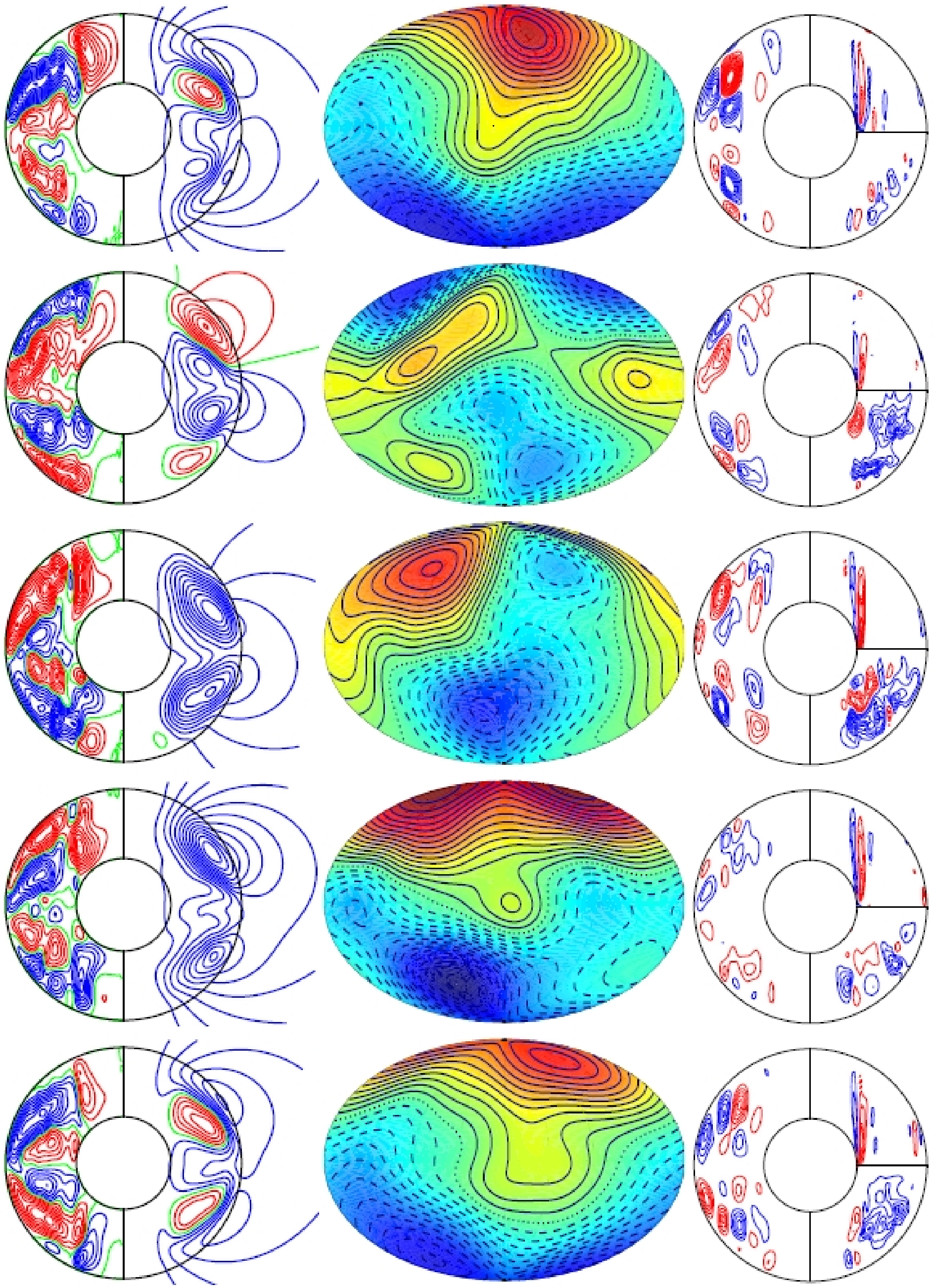
Fluctuating Dipolar (FD)

(b) ratio of kin energy components of **FD** to **MD** dynamos:

black.......**mean poloidal**
green......**fluctuating poloidal**
red..........**mean toroidal**
blue.........**fluctuating toroidal**

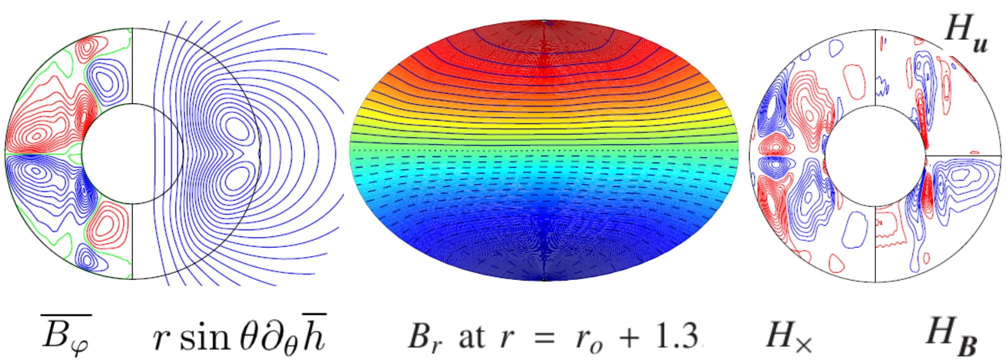
$$P = 0.75, \ P_m = 1.5$$
$$R \cdot 10^{-5} = 7.6, 17, 26, 35, 43, 51, 58, 62$$

Comparison of coexisting dynamos – magnetic features



$$R = 1.5 \times 10^6,\ \tau = 2 \times 10^4,$$
$$P = 0.75 \text{ and } P_m = 1.5$$

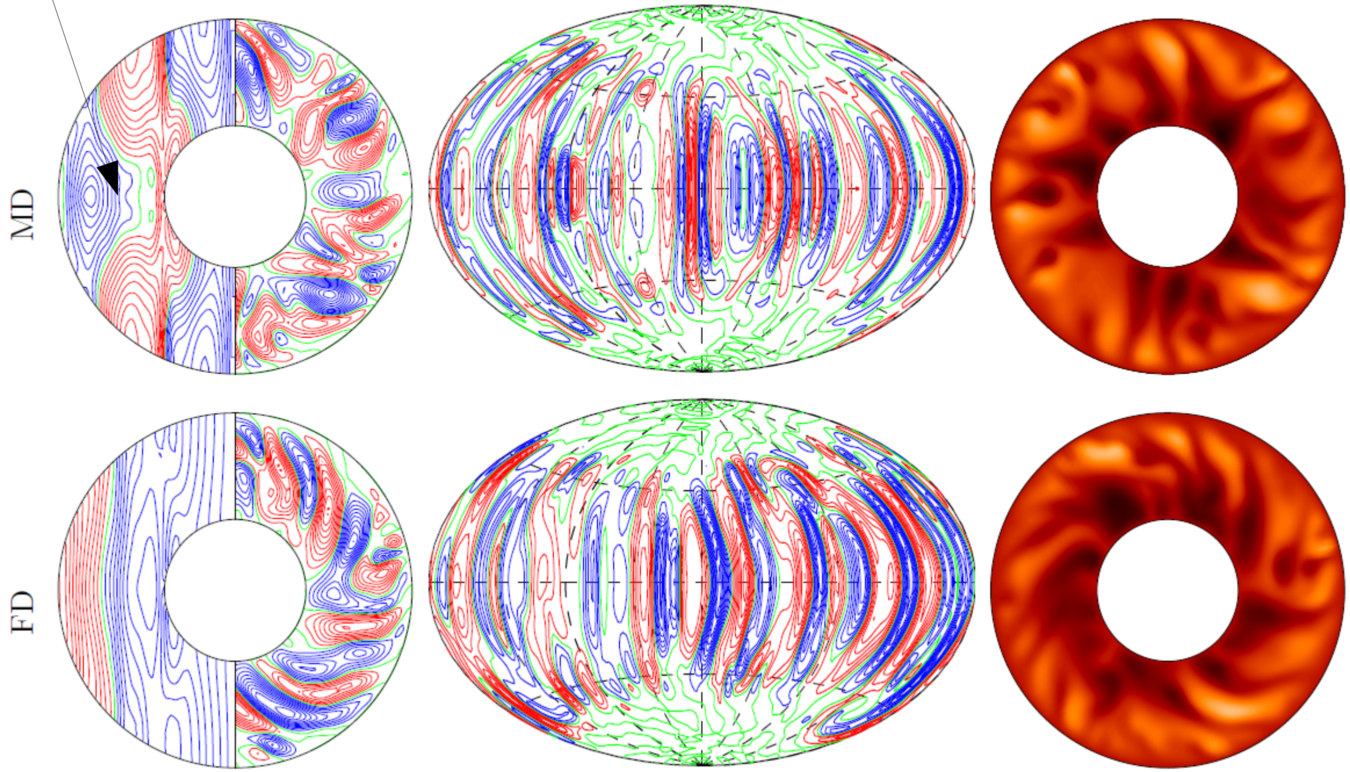
- ***Fluctuating Dipolar*** *(FD) dynamos are* ***oscillatory.***
- ***Mean Dipolar*** *(MD) dynamos are* ***non-oscillatory.***



Comparison of coexisting dynamos – convective features

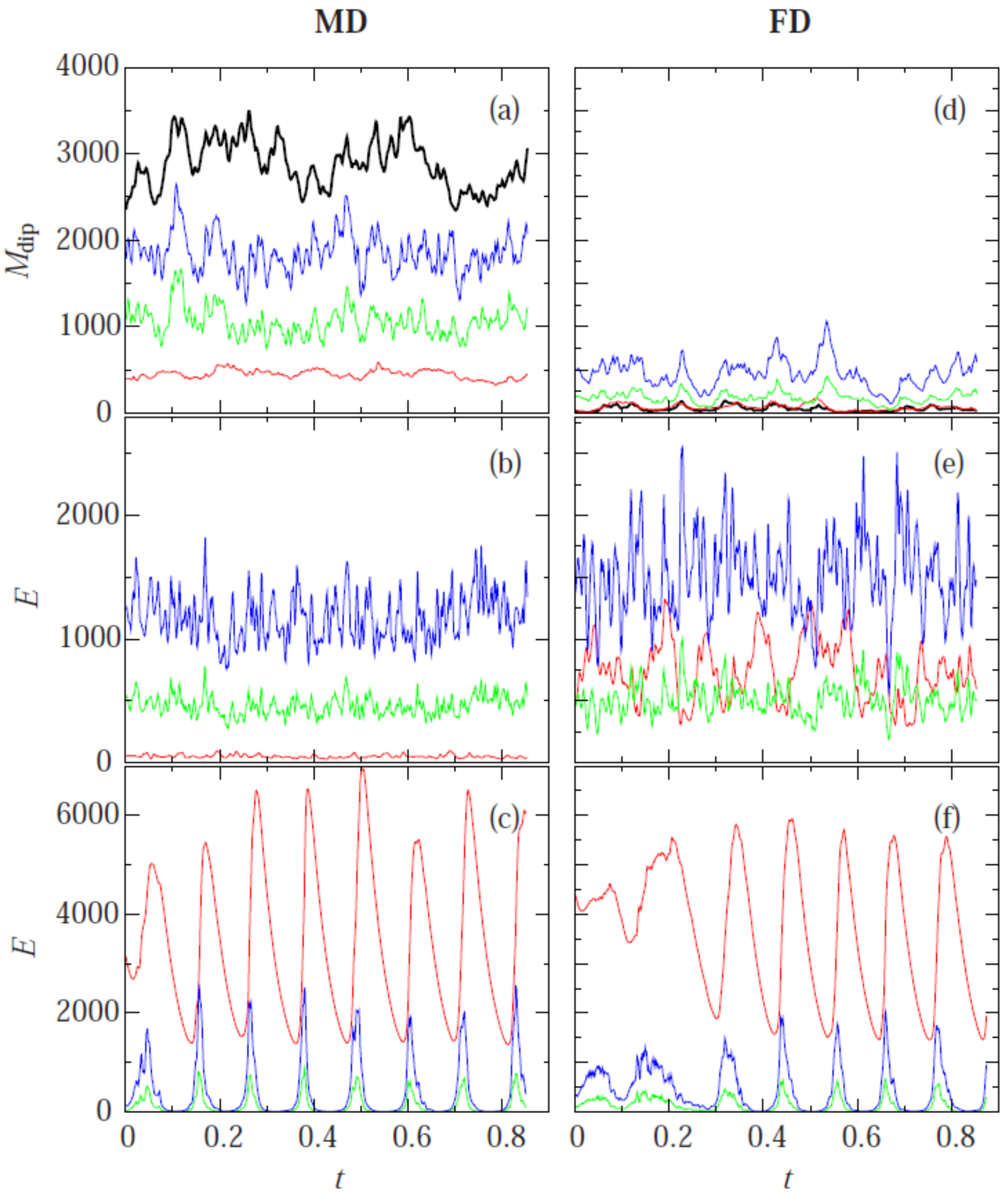
- *The stronger magnetic field of MD dynamos* ***counteracts*** *differential rotation*

$R = 1.5 \times 10^6$, $\tau = 2 \times 10^4$, $P = 0.75$ and $P_m = 1.5$



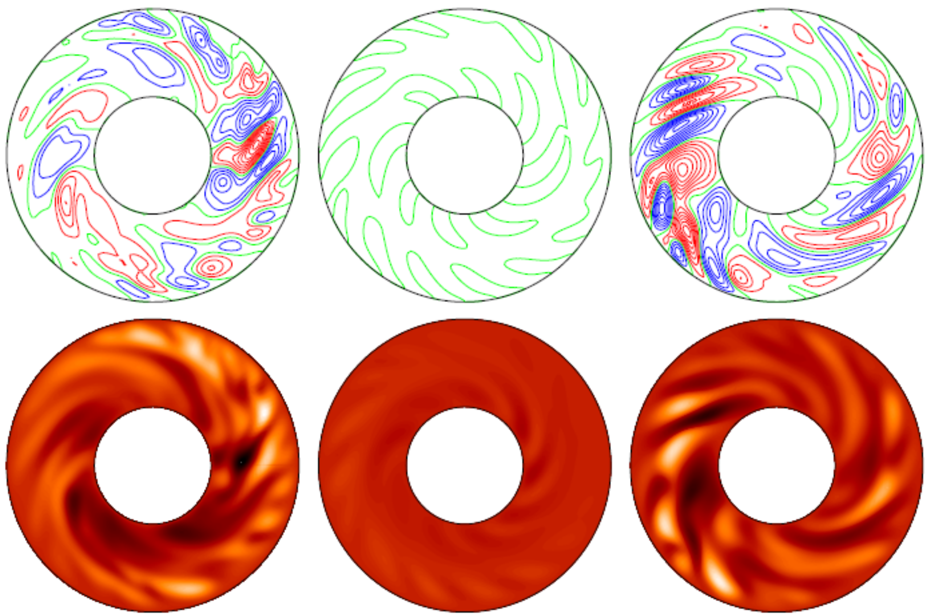
$\overline{u_\varphi}$ $r\partial_\varphi v$ u_r $(r = r_i + 0.5)$ Θ

The hysteresis is a purely magnetic effect



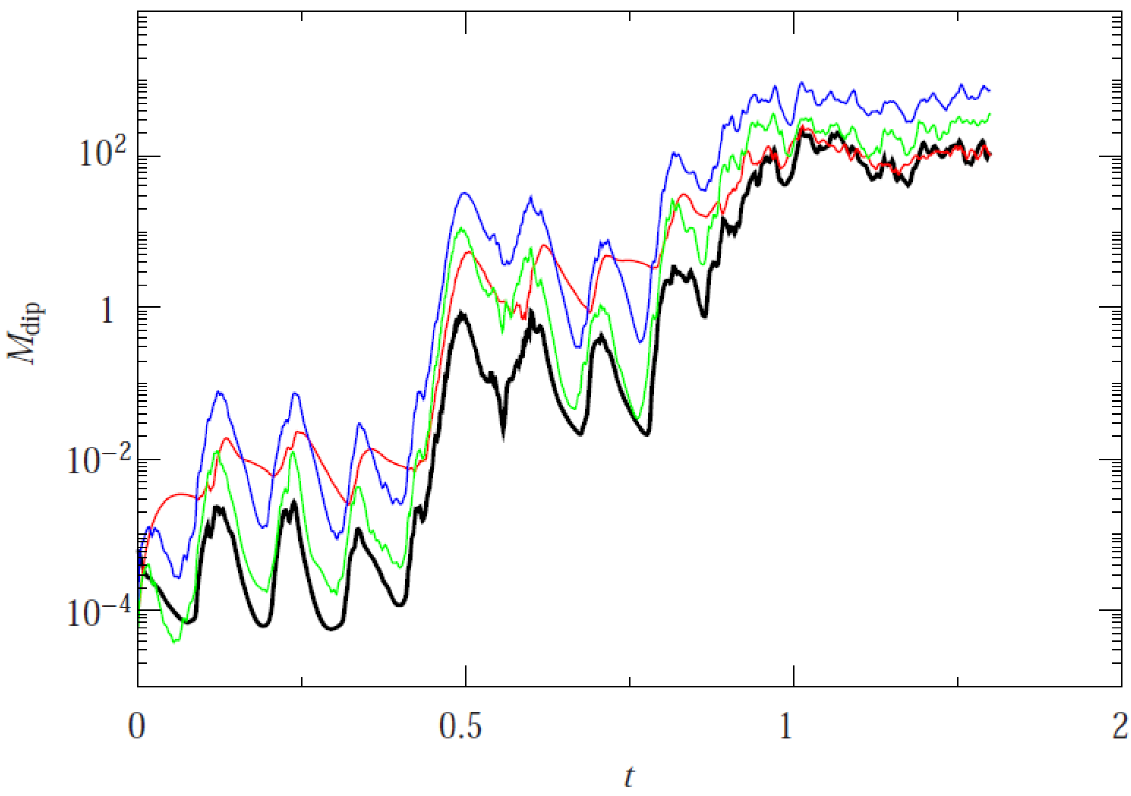
$R = 1.5 \times 10^6$, $\tau = 2 \times 10^4$, $P = 0.75$ and $P_m = 1.5$

- *After magnetic field is suppressed both MD and FD dynamos equilibrate to statistically **identical convective states** (period of relaxation oscillations, clockwise)*



*Magnetic field is artificially suppressed, i.e. **non-magnetic convection***

Basins of attraction – Random initial conditions



$R = 1.5 \times 10^6$, $\tau = 2 \times 10^4$,
$P = 0.75$ and $P_m = 1.5$

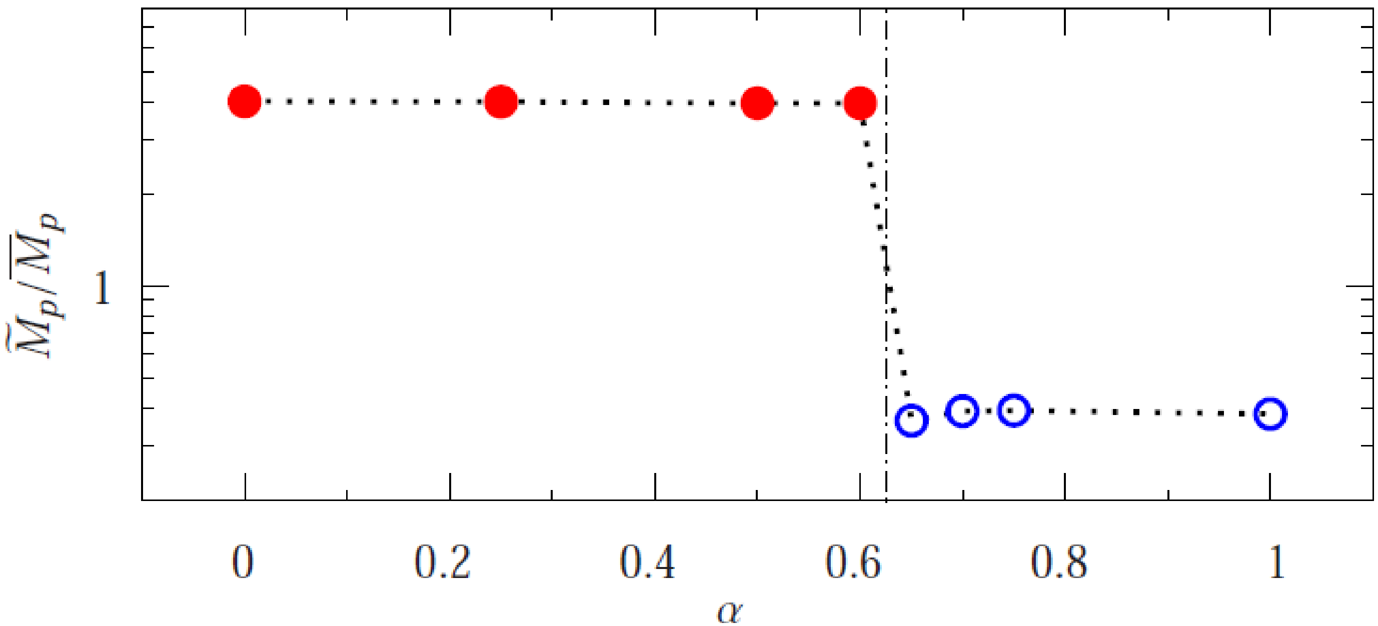
*black.......**mean poloidal***
*green......**fluctuating poloidal***
*red.........**mean toroidal***
*blue.........**fluctuating toroidal***

- *From small random initial seed the **Fluctuating Dipolar** state is approached.*
- *This, however means only that a the existence of a **third attractor is unlikely.***

Basins of attraction – Controlled initial conditions

- ***Initial conditions*** *– taken as a linear combination of* ***MD*** *and* ***FD*** *dynamos with a continuation parameter* $\alpha \in [0,1]$:

$$x(r,\theta,\varphi) = \alpha\, x^{\mathrm{FD}}(r,\theta,\varphi) + (1-\alpha)\, x^{\mathrm{MD}}(r,\theta,\varphi),$$



● ***Fluctuating Dipolar (FD)*** ○ ***Mean Dipolar (MD)***

$R = 1.5 \times 10^6$, $\tau = 2 \times 10^4$, $P = 0.75$ and $P_m = 1.5$

Part 2.

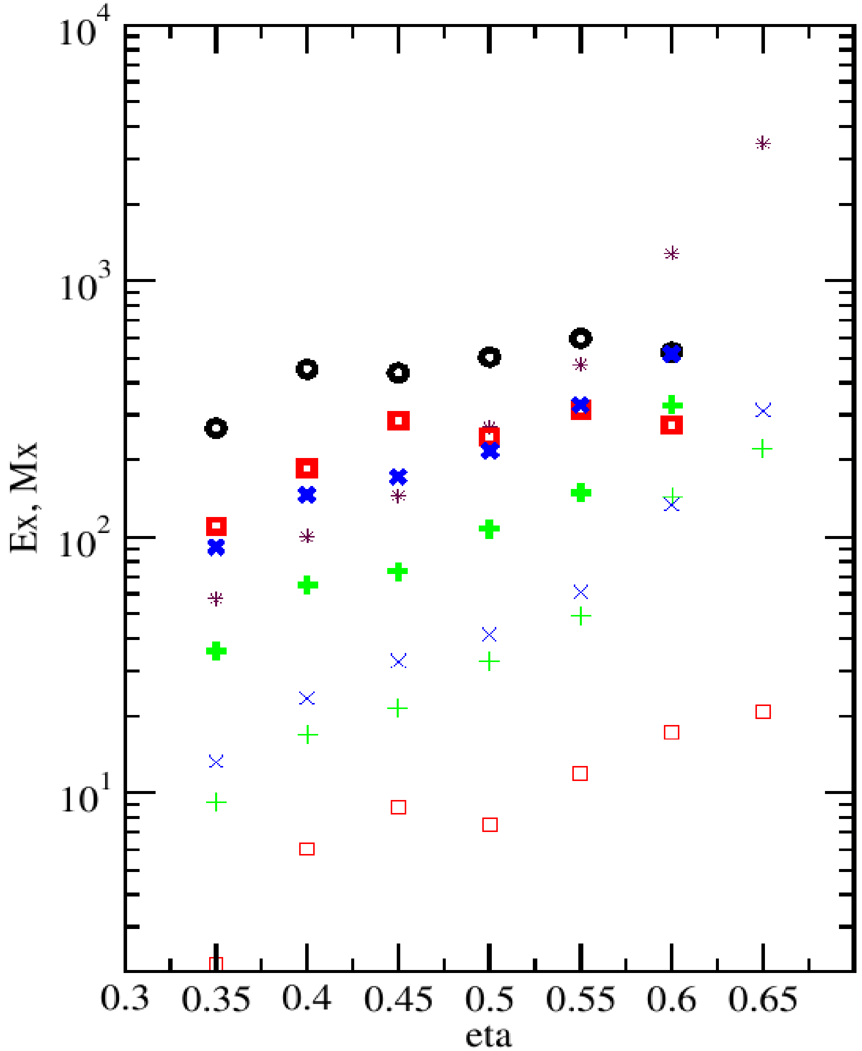
Minimal models of the solar cycle

- *Simitev, R., Busse F.H., Solar cycle properties described by simple convection-driven dynamos, **Physica Scripta** (accepted 13 Jan 2012).*
- *Simitev, R., Busse F.H., How far can minimal models explain the solar cycle?, **Astrophys. J.** (submitted 13 Jan 2012).*

Dependence on the shell thickness

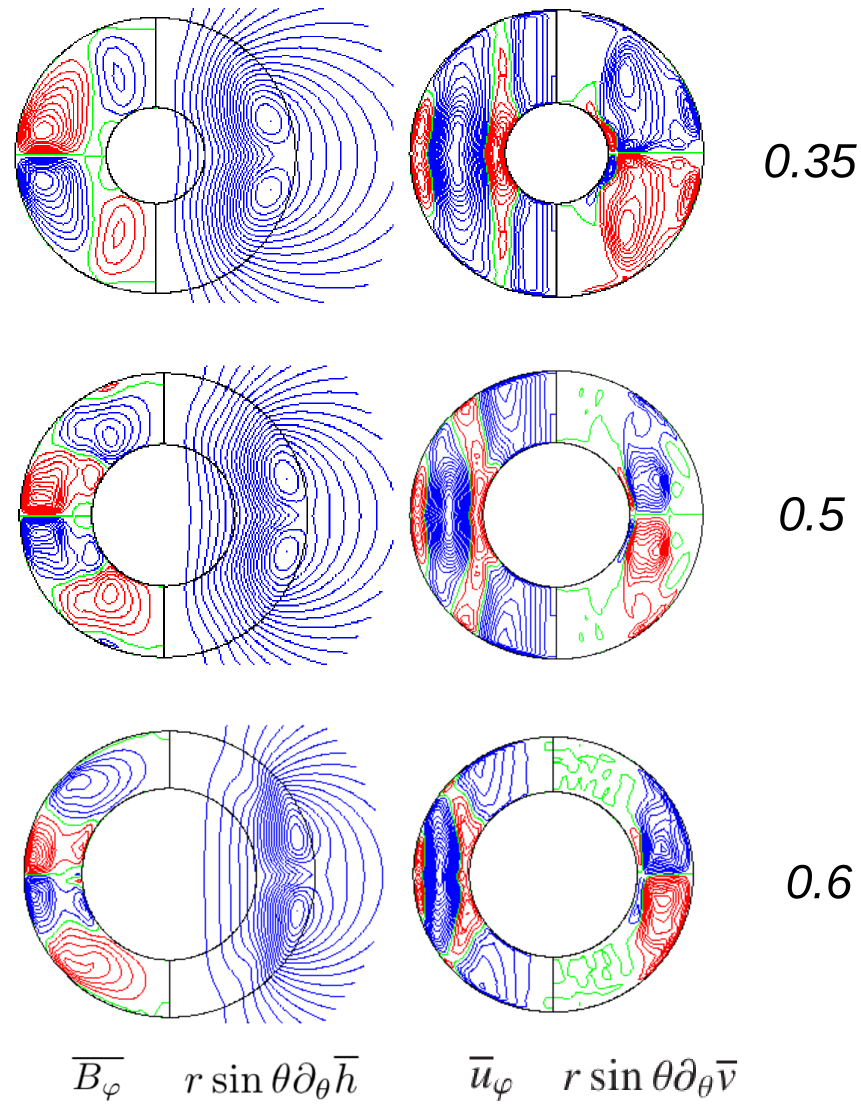
$P = 1, \tau = 2000, R = 10^5, P_m = 5$

no-slip boundary conditions

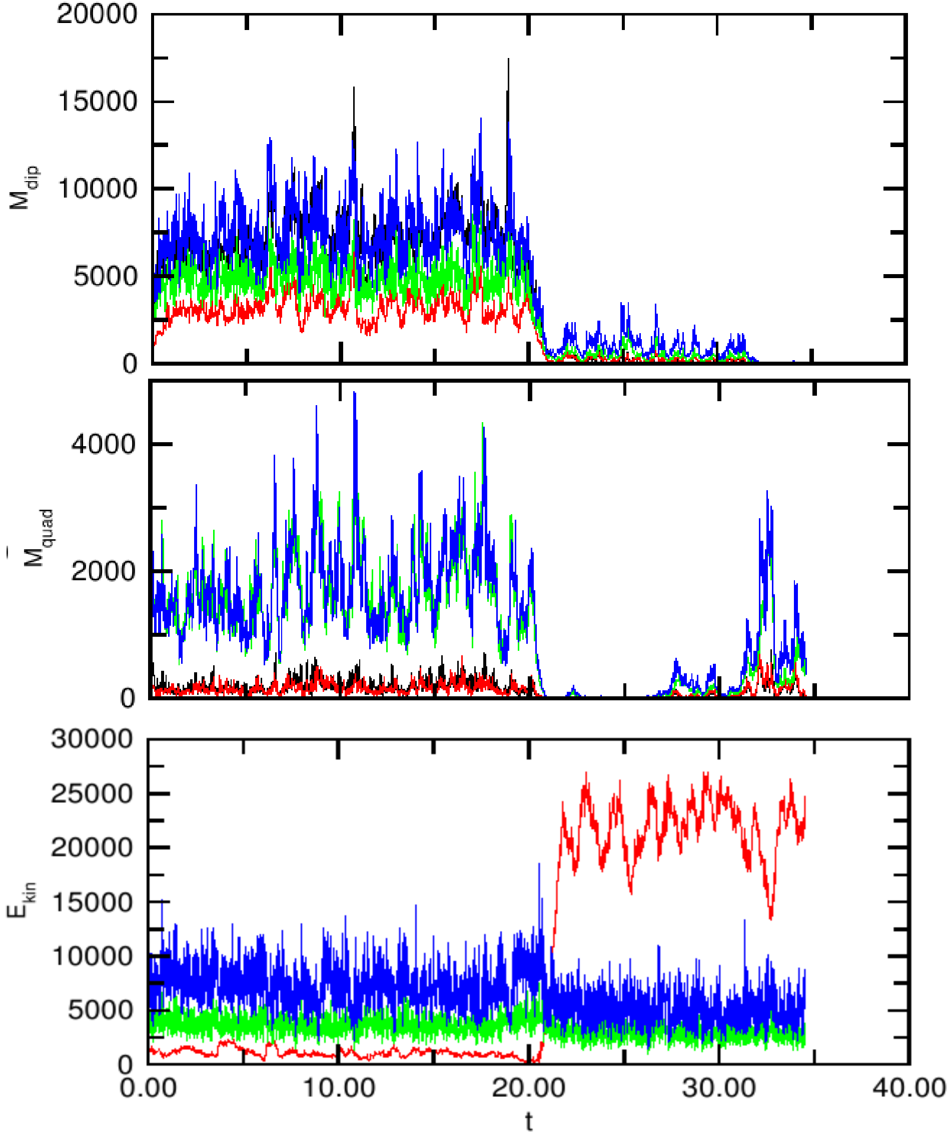


black.......**mean poloidal**
green......**fluctuating poloidal**
red..........**mean toroidal**
blue.........**fluctuating toroidal**

Motivation: *Goudard & Dormy, (EPL 2010)*



Transition to fluctuating dynamos in thin shells



Crucial assumption:

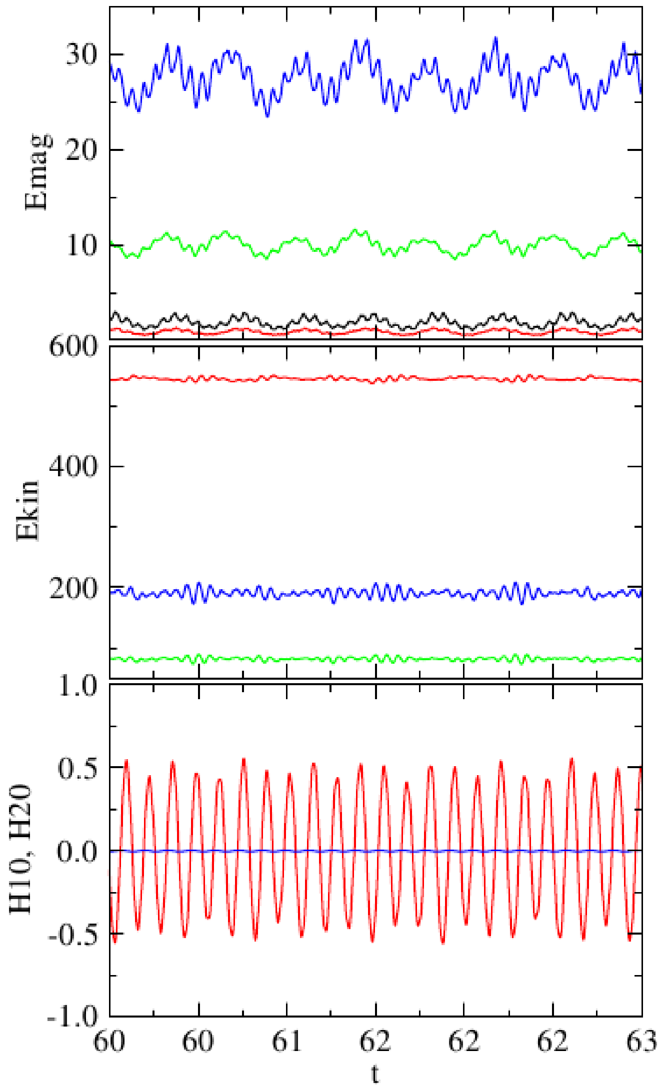
- ***Stress free at outer boundary***
- *No-slip at inner boundary*

$\eta = 0.65, P = 1, \tau = 2000,$

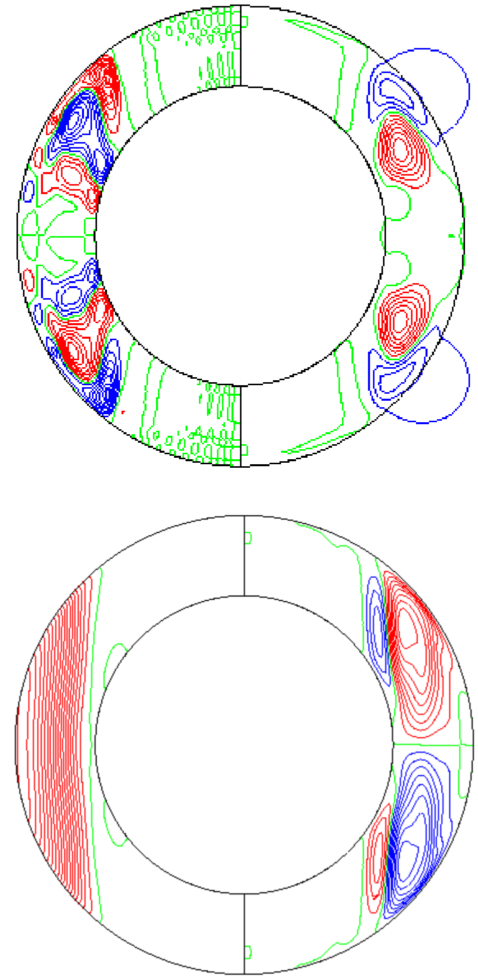
$R = 1.5 \times 10^5, P_m = 5$

*black.......**mean poloidal***
*green......**fluctuating poloidal***
*red.........**mean toroidal***
*blue........**fluctuating toroidal***

Regular dipolar oscillations in thin shells



$\overline{B_\varphi}$



$r \sin\theta \partial_\theta \overline{h}$

$\overline{u_\varphi}$

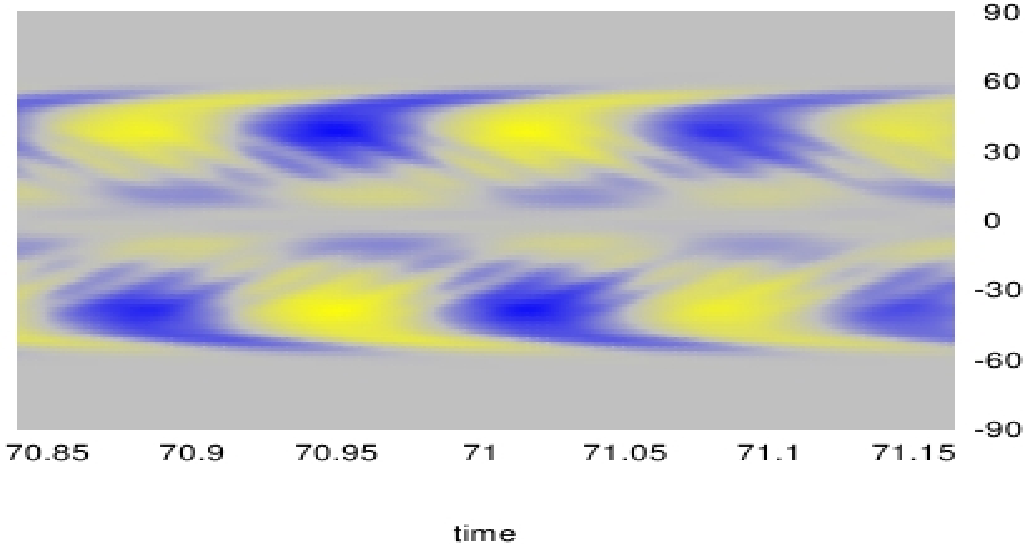
$r \sin\theta \partial_\theta \overline{v}$

$\eta = 0.65 \quad \mathrm{P} = 1,\ \mathrm{R} = 10^5,\ \tau = 2000,\ \mathrm{P_m} = 4.5,$

No-slip at inner boundary, stress free at outer boundary

Butterfly diagram

- *Not realistic – propagation if structures is in the opposite direction*

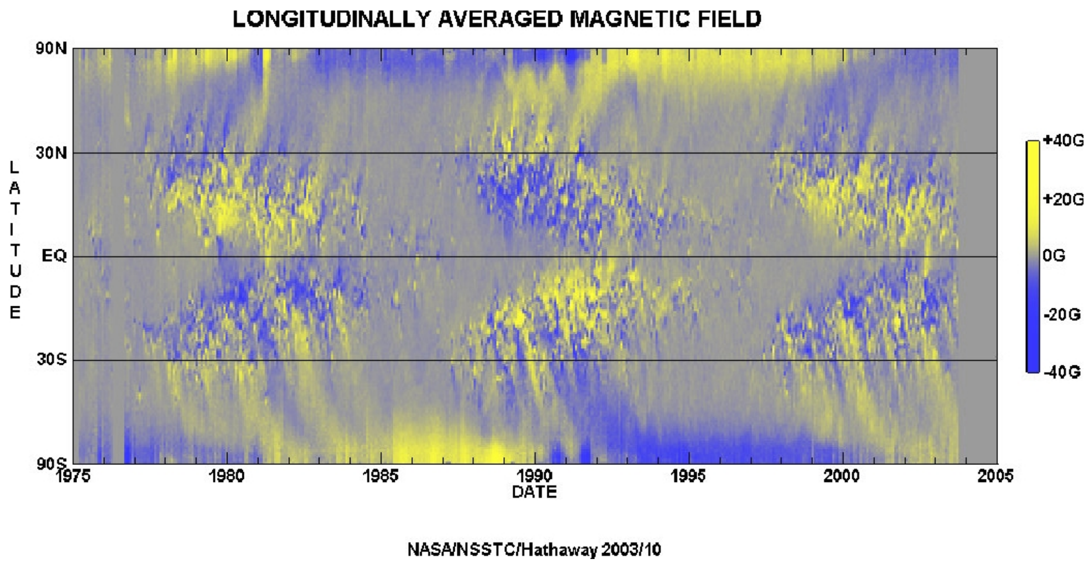


$$r\partial_\theta \overline{g} \quad \text{at } r = r_i + 0.95$$

$$\mathrm{P} = 1,\ \mathrm{R} = 10^5,\ \tau = 2000,$$

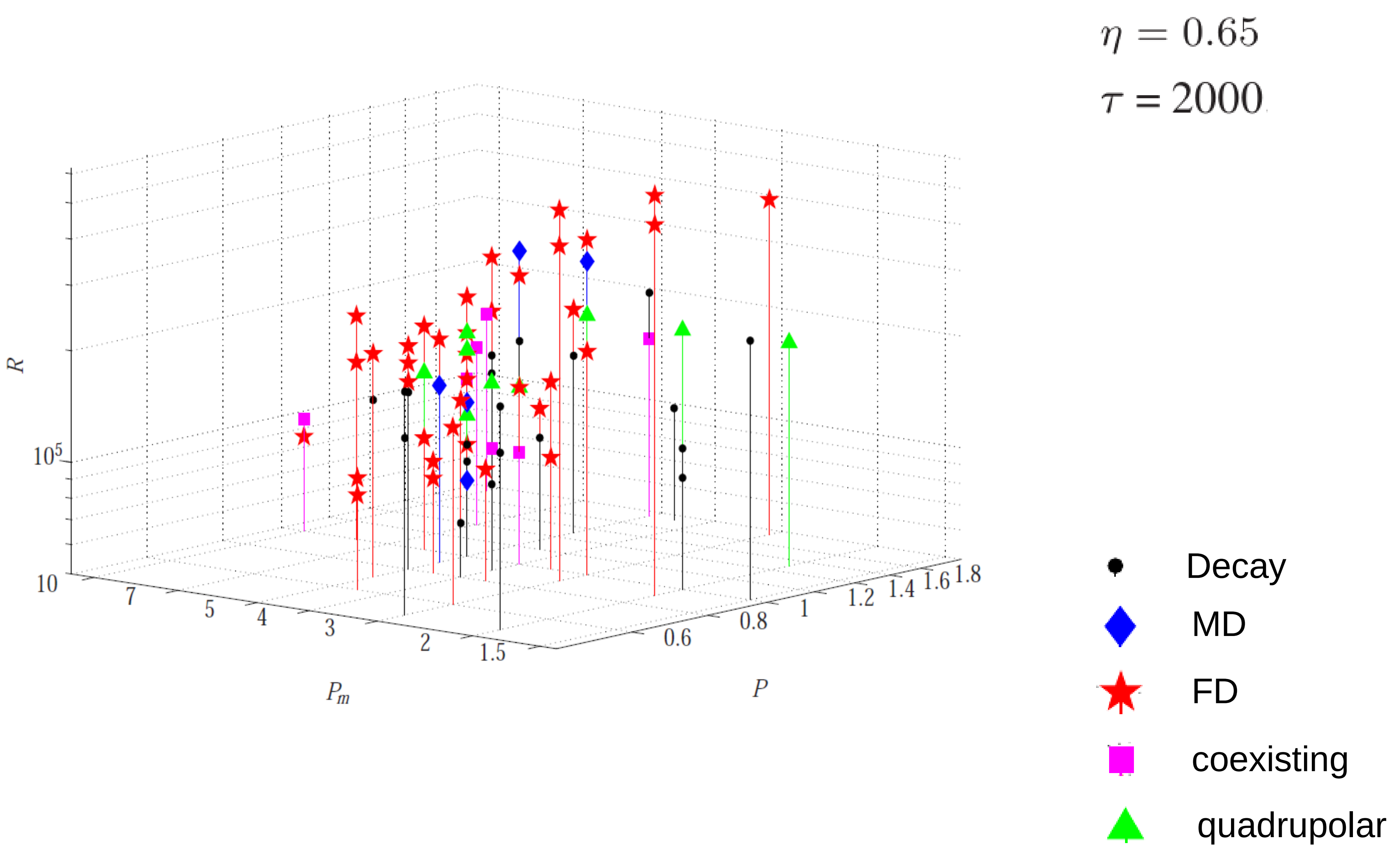
$$P_m = 4 \qquad \eta = 0.65$$

No-slip at inner boundary, stress free at outer boundary



Hathaway, D.H., et al, 2003, Astrophys. J., 589, 665-670

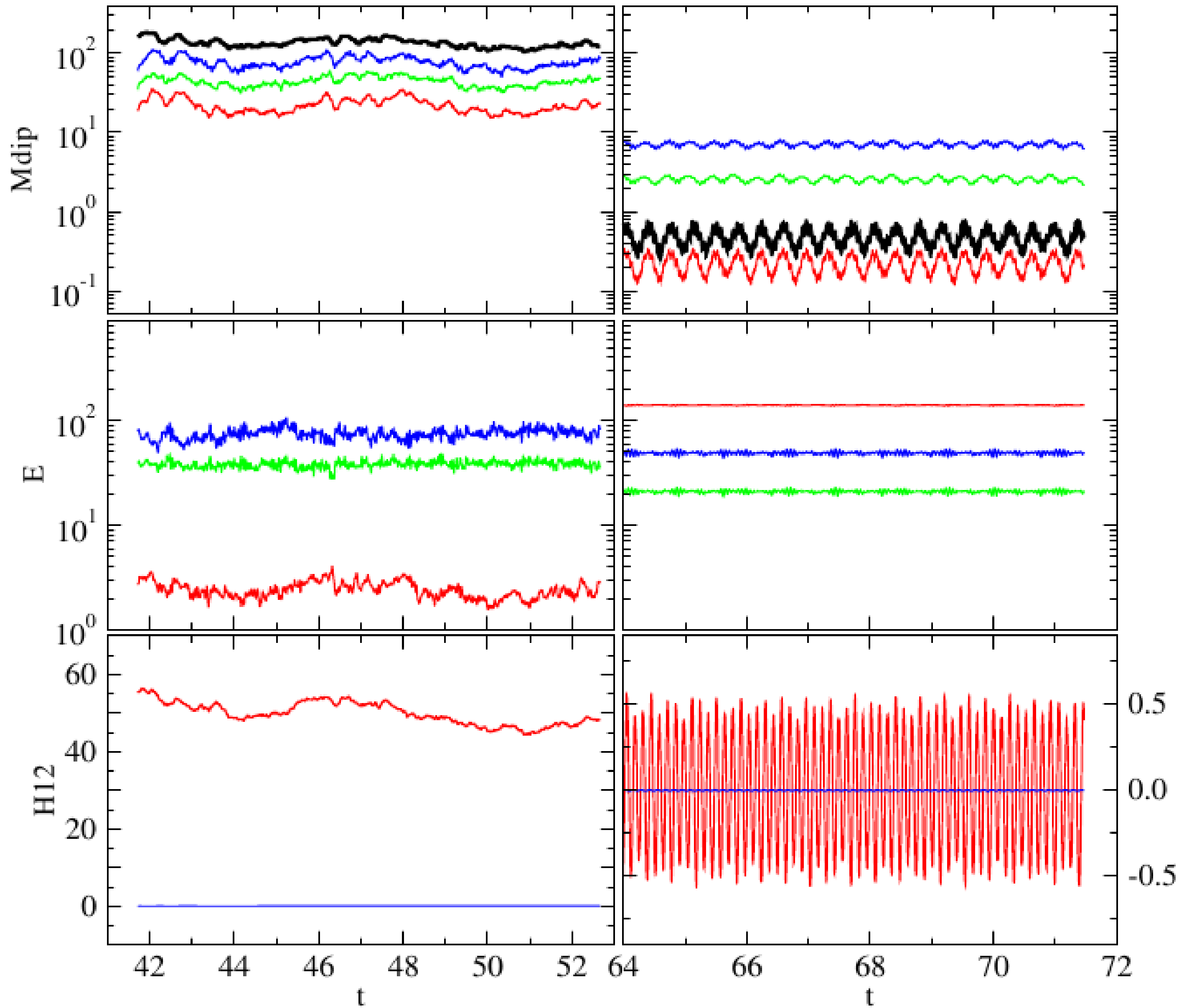
Thin-shell dynamos in the parameter space



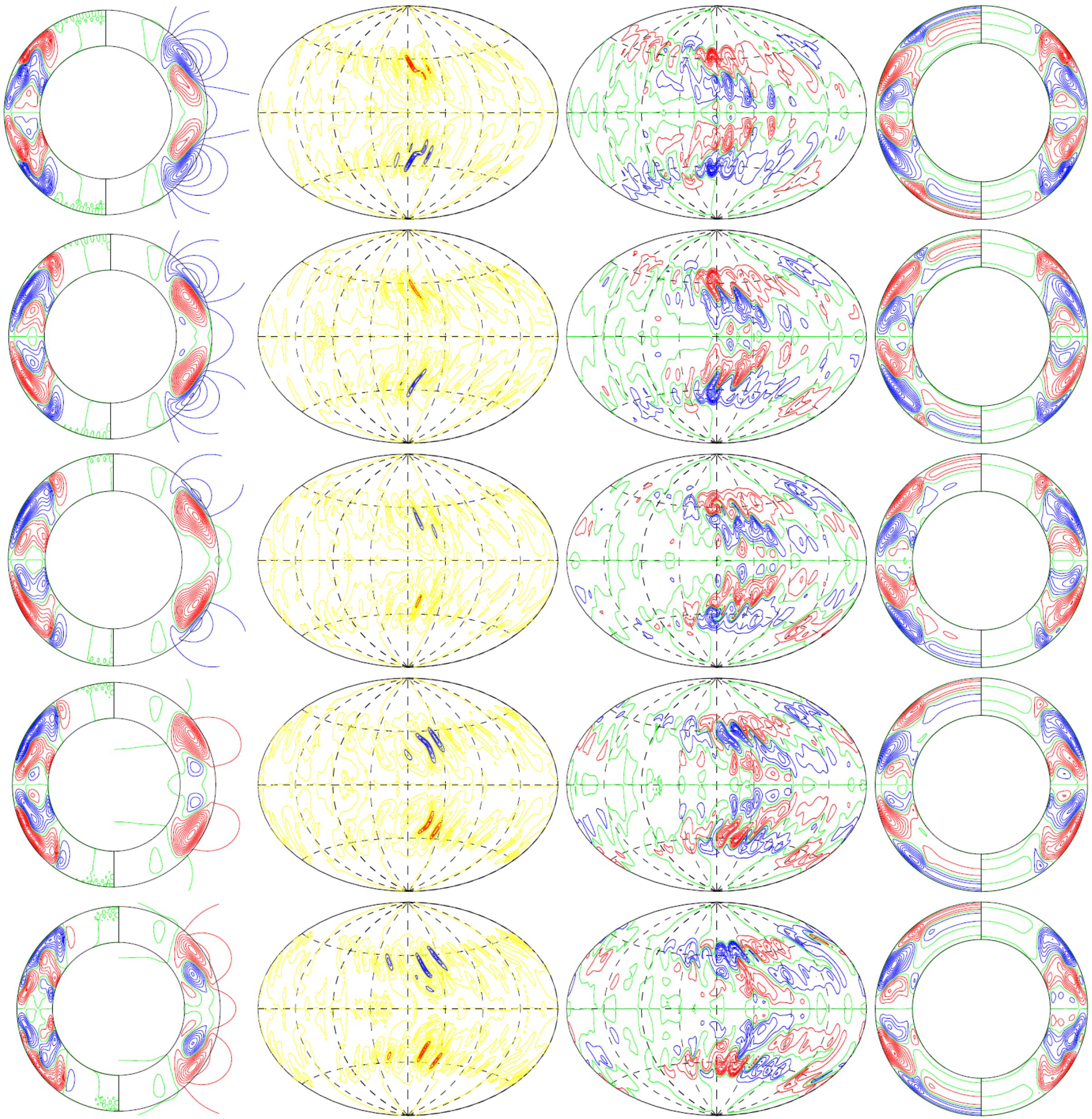
Bistability of dynamo solutions $\eta = 0.65$

$P = 1,\ R = 10^5,\ \tau = 2000,\ P_m = 4.5,$

No-slip at inner boundary,
stress free at outer boundary



m=1 azimuthal structure



The **m=1 structure**

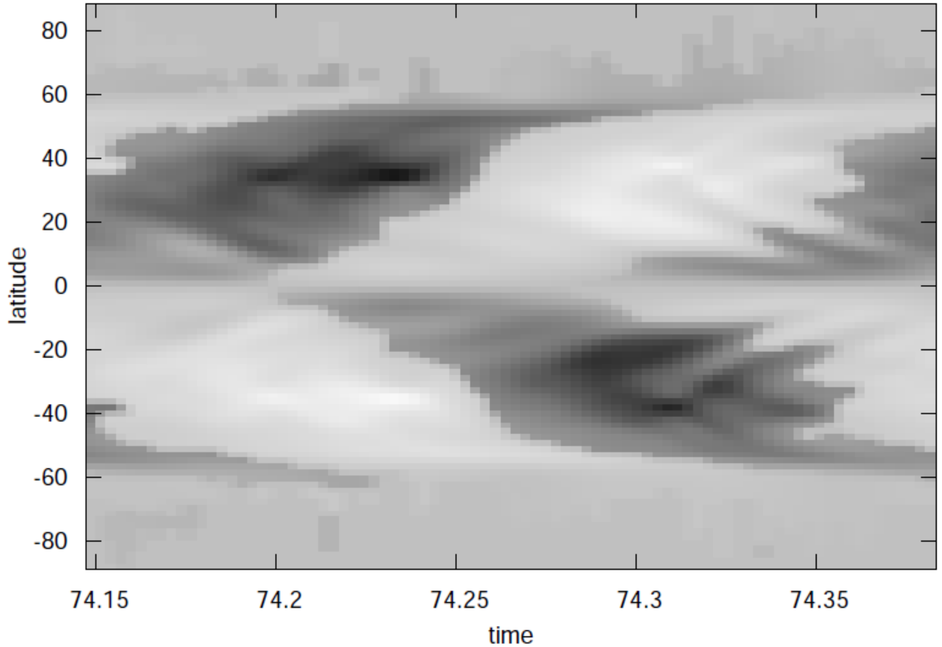
- resembles the phenomenon of **Active Longitudes** (Usoskin et al, Adv Space Res, 2007).
- suggests that the solar field might not be azimuthally symmetric as often assumed.

$\tau = 2000,\ R = 120000,$
$P = 1.2,\ P_m = 4.5.$

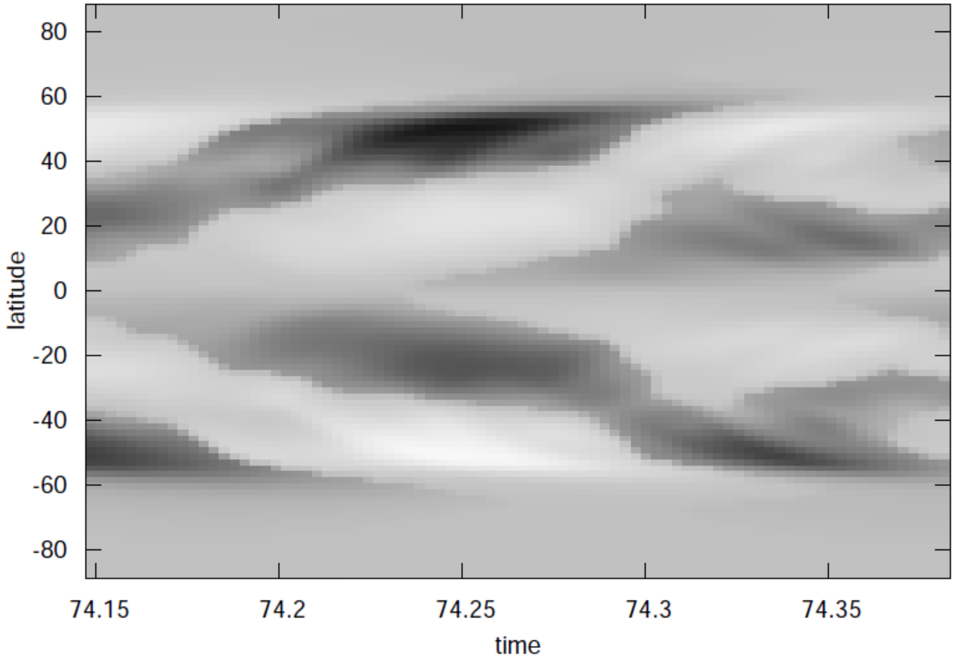
m=1 azimuthal structure – butterfly diagrams

- *Still unrealistic - no drift towards equatorial region*

$$B_\varphi^{m=0} + |B_\varphi^{m=1}|\,\mathrm{sgn}(B_\varphi^{m=0})$$



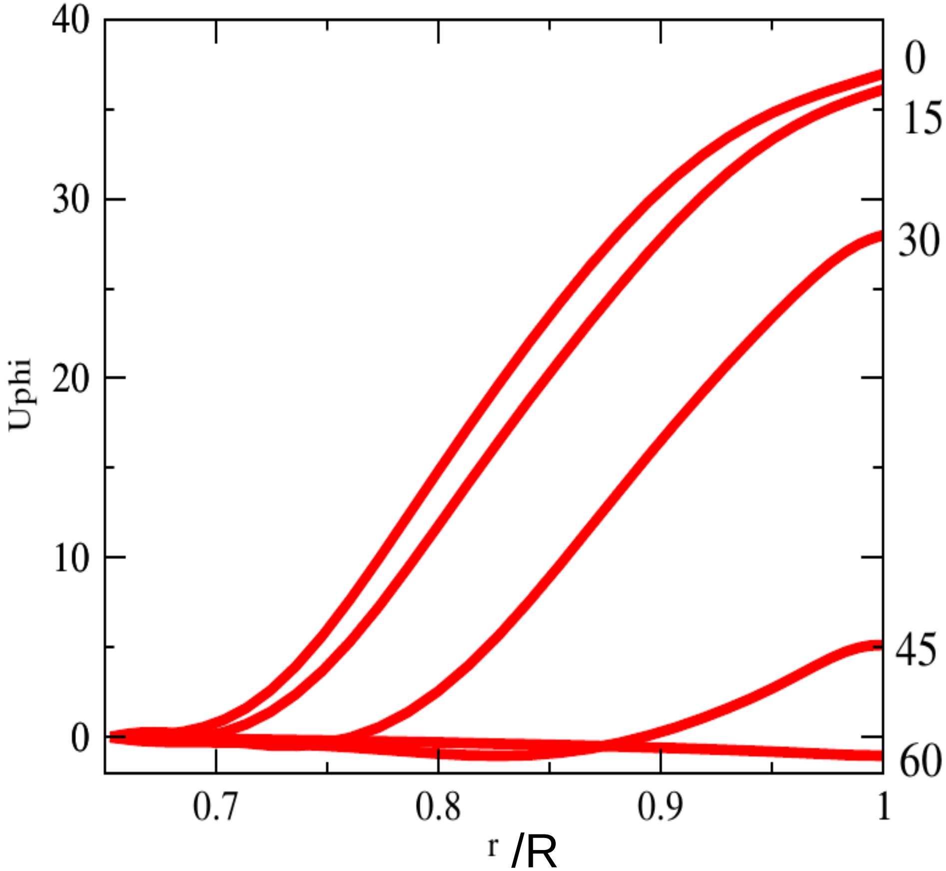
$$B_r^{m=0} + |B_r^{m=1}|\,\mathrm{sgn}(B_r^{m=0})$$



$$\tau = 2000,\ R = 120000, \quad P = 1.2,\ P_m = 4.5.$$

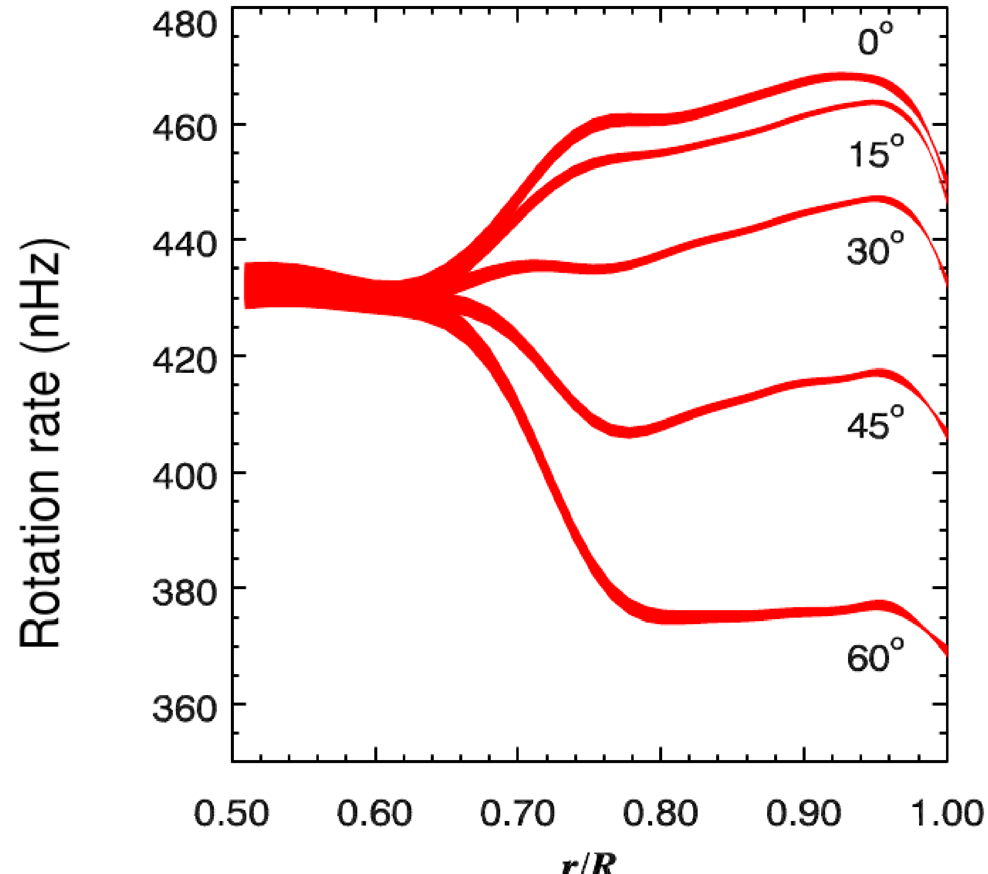
Radial profiles of the differential rotation

- *Unrealistic drift is due to the increasing profile of the differential rotation (Yoshimura, ApJ, 1975)*



$P = 1,\ R = 10^5,\ \tau = 2000,$
$P_m = 4 \qquad \eta = 0.65$

No-slip at inner boundary,
stress free at outer boundary



Internal Rotation of the Sun
as found by helioseismology,
NSF's National Solar Observatory

A modified boundary condition

- *Along with solar observations, a recent analytical analysis of a simplified model problem (Busse, Solar Physics 2007) indicates that a suitable way to reproduce the decrease of differential rotation with radius near the surface is to employ the following boundary condition*

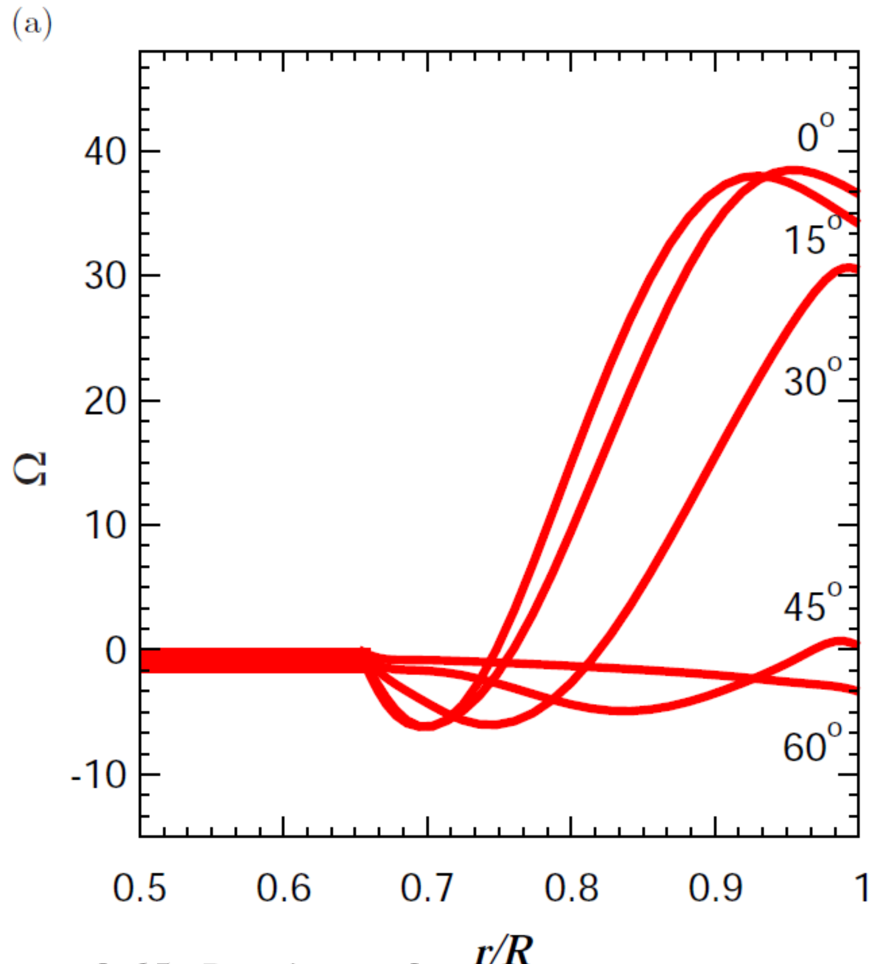
$$(\partial_r + \beta) W_{l=1}^{m=0} = 0$$

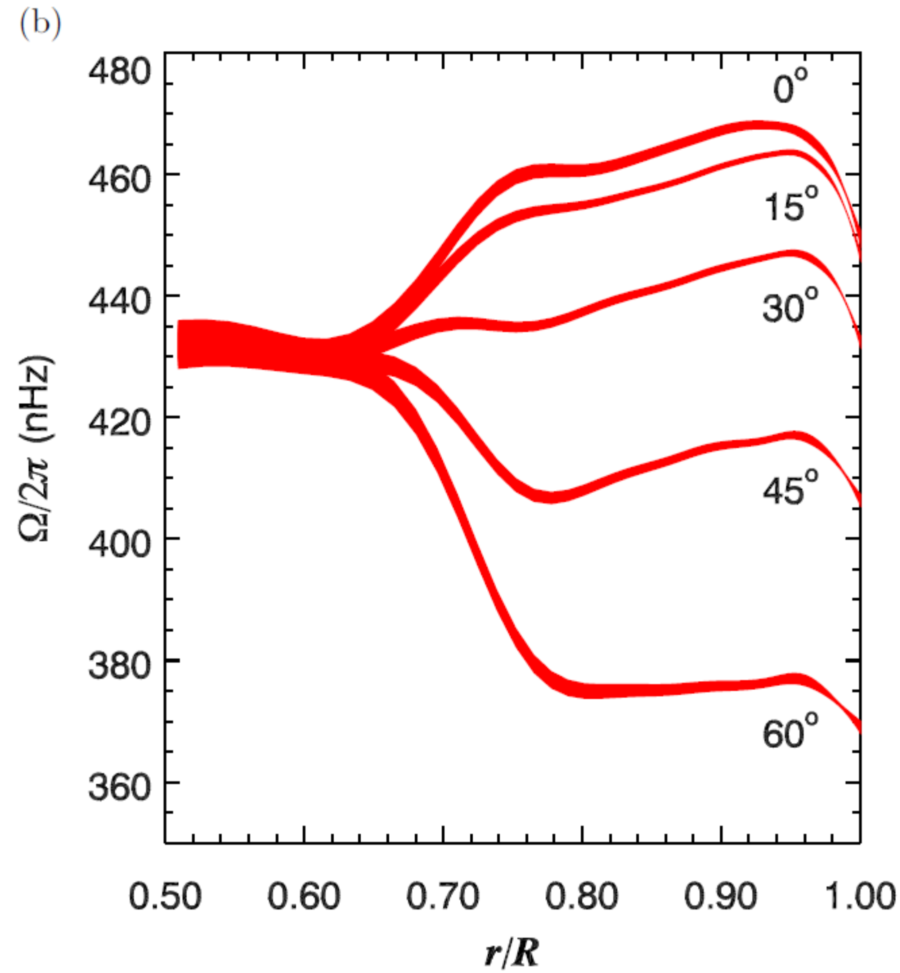
$$\text{at} \quad r = r_o$$

Beta is a fitting parameter.

Radial profiles of the differential rotation

- *Profile is still unrealistic but decreasing near the surface.*





$\eta = 0.65, P = 1, \tau = 2,$
$R = 120000, P_m = 4, \beta = 1.5$

No-slip at inner boundary,
stress free at outer boundary

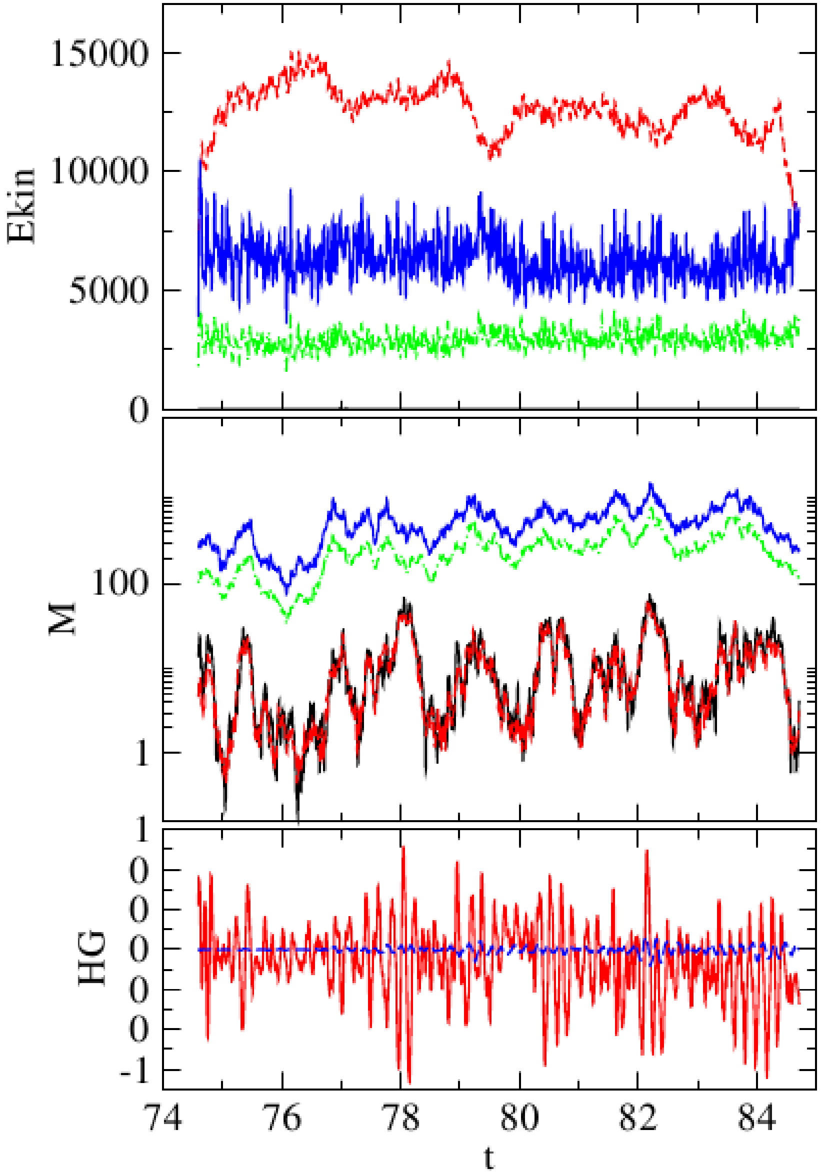
Internal Rotation of the Sun
as found by helioseismology,
NSF's National Solar Observatory

Regularly oscillating dipoles with $\beta \neq 0$

After systematic variation of parameter values cases with regular oscillations have been found.

$$\beta = 0.5,\ \eta = 0.65,\ P = 1$$
$$\tau = 2000,\ R = 1.2 \times 10^5$$
$$P_m = 4$$

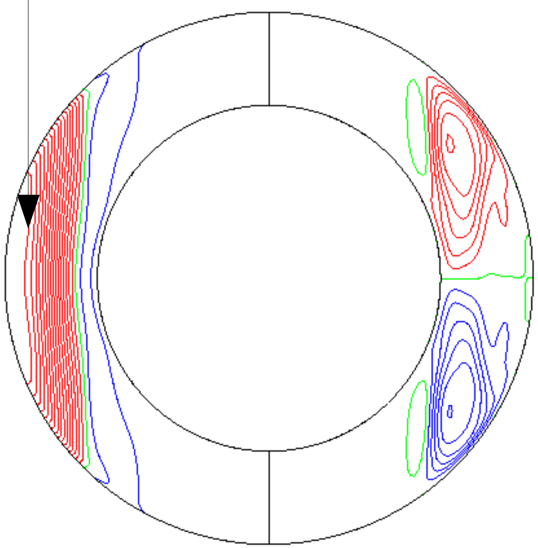
Fairly regular dipolar oscillations →

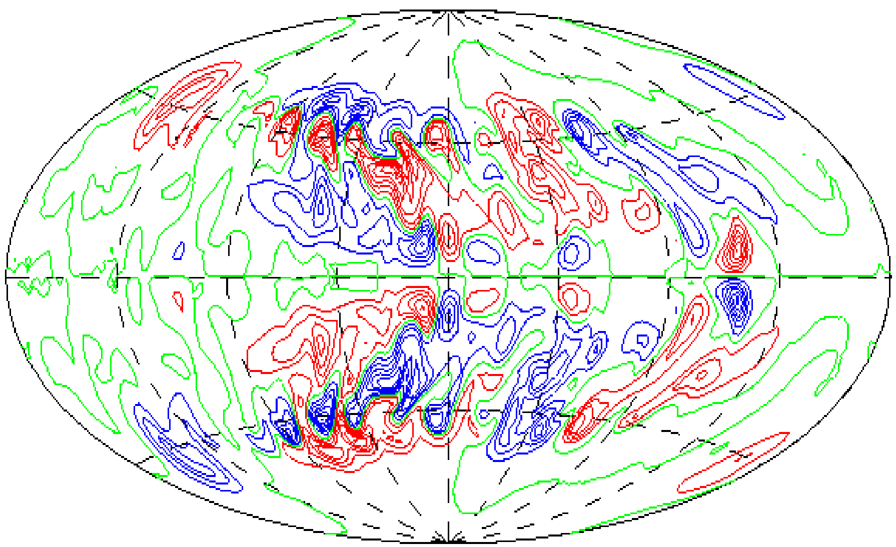


Sunspot drift in dipoles with $\beta \neq 0$

Diff rot decreases near surface
near surface

Active longitudes – m=1 structure



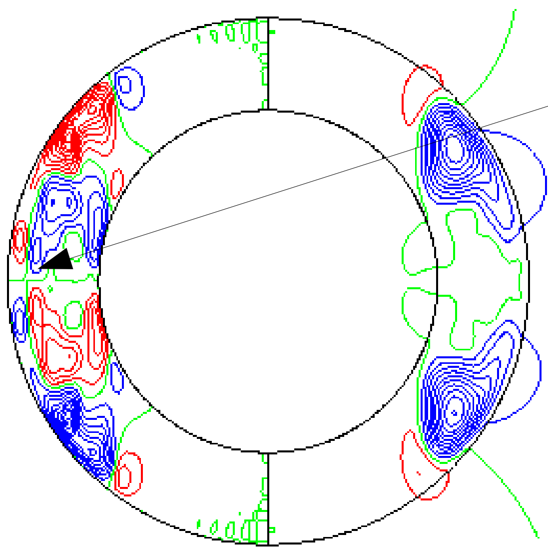


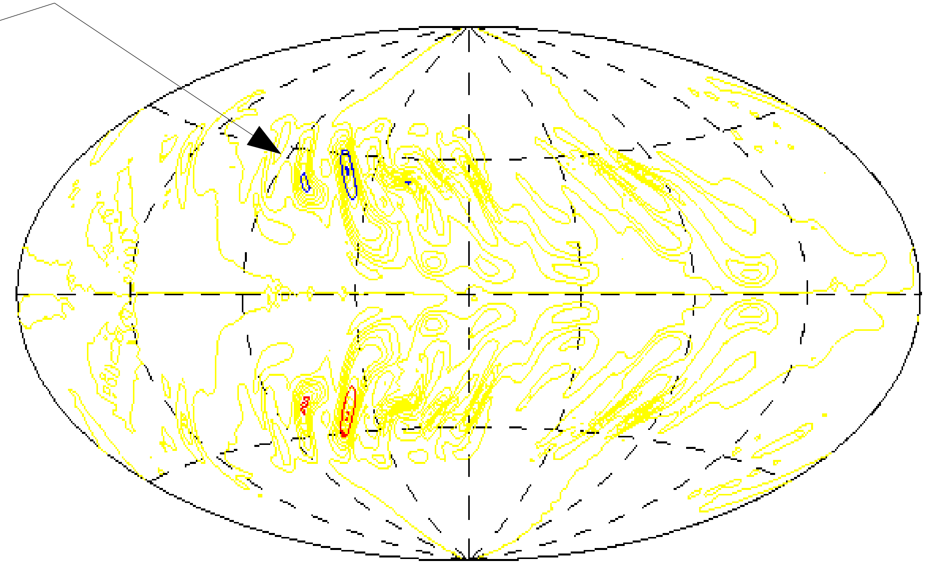
$\overline{u_\varphi}$ $\quad r \sin\theta \partial_\theta \overline{h}$ $\quad B_r$ at $r = r_o$

$\beta = 1.5,\ \eta = 0.65,\ P = 1 \quad \tau = 2000,\ R = 1.2 \times 10^5 \quad P_m = 4$

Sunspot drift in dipoles with $\beta \neq 0$

Maxima of Bphi propagate from higher latitudes towards the equator





$\overline{B_\varphi}$ $\quad r \sin\theta \partial_\theta \overline{h}$ $\quad \partial_\theta g \quad \text{at } r = r_i + 0.98$

$\beta = 1.5,\ \eta = 0.65,\ P = 1 \quad \tau = 2000,\ R = 1.2 \times 10^5 \quad P_m = 4$

Conclusion

Part A

- *Bistability seems to occur in a certain region as a function of all parameters.*
- *Many published dynamo simulations are within the region of bistability – special care is needed.*

Part B

- *The large scale Solar dynamo might be dominated by a non-axisymmetric m=1 component of the magnetic field.*
- *Periodically reversing FD dynamos in thin shells with an equator-ward drift of magnetic structures may be a good minimal model of the solar cycle.*