2A Multivariable Calculus 2013

Ex Sheet 3

Tutorial Exercises

T1 By making the change of variables indicated, find the general solution of each of the following partial differential equations.

a) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 6xy$. Change to $u = \frac{y}{x}$ and $v = x$

b) $2x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y} = 2xy$. Change to $u = xy^2$, and $v = y$

T2 Evaluate

$$\text{(a)} \int_0^1 dx \int_0^2 3y^2 - 4x\, dy, \quad \text{(b)} \int_0^1 dx \int_0^1 2x + 10y\, dy.$$

T3 Evaluate

$$\text{(a)} \int_1^2 dx \int_1^x \frac{1}{x+y}\, dy, \quad \text{(b)} \int_0^{\pi/2} dy \int_y^4 x \sin y\, dx.$$

T4 Sketch the triangular domain T, bounded by the lines $y = -x$, $y = 0$ and $x = 1$ and illustrate that it is both type I and type II. Evaluate the double integral

$$\iint_T x\, dxdy,$$

using (a) the type I formulation of T and (b) the type II formulation of T.[1]

[1] The answers you get to (a) and (b) should, of course, be the same.

T5 Evaluate

$$\iint_D e^{x+y}\, dxdy,$$

where D is the triangle with vertices $(0,0)$, $(1,1)$ and $(-1,1)$.

Further Exercises

F1 By making the change of variables indicated, find the general solution of each of the following partial differential equations.

a) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3y(y^2 - x^2)$. Change to $u = x$, $v = \frac{y}{x}$.

b) $2x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{6x^4}{y^2}$. Change to $u = \frac{x}{y^2}$, and $v = x$

c) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{4x^3}{y}$. Change to $u = \frac{x}{y}$, $v = x$.

Lecture 5

- **Key Points:**
  - finding the general solution to a PDE using change of variables
  - definition of a double integral
  - calculating double integrals over rectangular domains
- **Read:**
  - Stewart Section 15.1 (p997)
  - Stewart Section 15.2 (p1006)
- **Textbook Exercises:**
  - Exercises 15.2 (p1011) Qs 1–31

Lecture 6

- **Key Points:**
  - definition of type I, type II and regular domains
  - calculating double integrals over regular domains
- **Read:**
  - Stewart Section 15.3 (p1012)
- **Textbook Exercises:**
  - Exercises 15.3 (p1019) Qs 1–32

F2 Evaluate

$$\int\int x^2 + 2y \, dxdy$$

over the rectangle with vertices at $(0,0)$, $(2,0)$, $(2,3)$ and $(0,3)$.

F3 Evaluate

$$\int\int xy \, dxdy$$

over the triangle enclosed by the lines $y = 2x$, $y = 4$ and the y-axis.

F4 Evaluate

$$\iint_D xy \, dxdy,$$

where D is the finite region bounded by the curves $y = x^2$ and $x = y^2$.

F5 Sketch the tetrahedron T formed by the plane $x + 2y + 3z = 6$ and the xy-, xz- and yz-planes. Show that the volume of T is

$$V = \frac{1}{3}\iint_D 6 - x - 2y \, dxdy,$$

where D is the finite region bounded by $x = 0$, $y = 0$ and $x + 2y = 6$. Hence evaluate V.

F6 Evaluate

$$\int\int x \, dx \, dy$$

over the trapezium with vertices at $(0,0)$, $(4,0)$, $(3,1)$ and $(1,1)$.

F7 Evaluate

$$\int\int e^{-(x+y)} \, dx \, dy$$

over the region given by the inequalities $y \geq 0$, $y \leq 1$ and $y \leq x$.

[2] Harder challenge problems

[2] Only attempt these if you have been able to do all the other problems successfully.

F8 Find the volume of the given solid

a) Bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant.

b) Bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$.

F9 Use geometry or symmetry, or both, to evaluate the double integral

$$\iint_D (2 + x^2y^3 - y^2 \sin x) \, dA,$$

where $D = \{(x,y) | |x| + |y| \leq 1\}$.