2A Multivariable Calculus 2013

Ex Sheet 0

Differentiation

Q1 Find the derivatives of the following functions

a) $y = (2x+3)^6$ (Ans: $12(2x+3)^5$)

b) $y = \frac{3}{\sin(2x)}$ (Ans: $-6\text{cosec}(2x)\cot(2x)$).

c) $y = \sqrt{x+7}$ (Ans: $\frac{1}{2\sqrt{x+7}}$)

d) $y = x^3 \tan(x)$ (Ans: $3x^2 \tan(x) + \frac{x^3}{\cos^2 x}$).

e) $y = \frac{\ln x}{x}$ (Ans: $\frac{1-\ln x}{x^2}$)

f) $y = \cos(3x+2)$ (Ans: $-3\sin(3x+2)$).

g) $y = \frac{1+\sqrt{x}}{1-\sqrt{x}}$ (Ans: $\frac{1}{\sqrt{x}(1-\sqrt{x})^2}$)

h) $y = e^{x^2}$ (Ans: $2xe^{x^2}$).

i) $y = \sec(\sqrt{x}) + 3$ (Ans: $\frac{\sec\sqrt{x}\tan\sqrt{x}}{2\sqrt{x}}$).

j) $y = \sin^{-1} x$ (Ans: $\frac{1}{\sqrt{1-x^2}}$).

Integration

Q2

a) $\int xe^x\,dx$ (Ans: $xe^x - e^x + C$)

b) $\int \frac{x}{2x^2+5x+2}$ (Ans: $\frac{2}{3}\ln|x+2| - \frac{1}{6}\ln|2x+1| + C$).

c) $\int e^{-x}\sin(2x)\,dx$ (Ans: $-e^{-x}\left(\frac{2}{5}\cos 2x + \frac{1}{5}\sin 2x\right) + C$)

d) $\int \frac{3}{4x^2-1}\,dx$ (Ans: $\frac{3}{4}\ln\left|\frac{2x-1}{2x+1}\right| + C$).

e) $\int \frac{\cos x}{4+\sin^2 x}\,dx$ (Ans: $\frac{1}{2}\tan^{-1}\left(\frac{\sin x}{2}\right) + C$)

f) $\int xe^{-4x^2}\,dx$ (Ans: $\frac{-1}{8}e^{-4x^2} + C$).

g) $\int \sin x\cos x\,dx$ (Ans: $\frac{-1}{4}\cos(2x) + C$)

h) $\int_0^8 \frac{\cos(\sqrt{x+1})}{\sqrt{x+1}}\,dx$ (Ans: $2\sin 3 - 2\sin 1$).

i) $\int \ln x\,dx$ (Ans: $x\ln x - x + C$).

j) $\int \frac{2+3x+x^2}{x(x^2+1)}\,dx$ (Ans: $2\ln x + 3\tan^{-1} x - \frac{1}{2}\ln(x^2+1) + C$).

Pre-course Revision

You should be very familiar and have a good command of all the course pre-requisites *before* the start of the course next week. On this sheet you will find exercises from 1R and 1S, try a selection of these problems and seek out help (from your peers, lecturers, *etc.*) if you are struggling in any way — this course builds assumes a good knowledge of this material and you run the risk of loosing your way if you are not proficient!

Course Prerequisites

We will assume that everyone is familiar with basic material on differentiation, integration and vectors. This includes:

- differentiation
  - product rule,
  - quoitent rule,
  - chain rule,
  - implicit differentiation.
- integration,
  - method of substitution,
  - integration by parts,
  - partial fractions and integrals of rational functions
- vectors in $\mathbb{R}^3$
  - manipulating expressions involving vector notation
  - scalar product and it's properties (e.g. the scalar product of two non-zero vectors is 0 if and only if the vectors are perpendicular,
  - vector product and it's properties
  - triple scalar product and triple vector product

A summary of some of this key material from the Level 1 courses has been posted on the 2A Moodle page for convenience, but you are also strongly encouraged to review your Level 1 lecture notes.

Vector algebra

Q3 Let $\mathbf{a} = (1, 5, 3)$, $\mathbf{b} = (2, 4, 7)$, $\mathbf{c} = (2, 0, -1)$. Find

$$\mathbf{a} \cdot \mathbf{b}, \quad \mathbf{a} \times \mathbf{b}, \quad \mathbf{b} \times \mathbf{c}, \quad [\mathbf{a}, \mathbf{b}, \mathbf{c}].$$

(Ans: $\mathbf{a} \cdot \mathbf{b} = 43$, $\mathbf{a} \times \mathbf{b} = (23, -1, -6)$, $\mathbf{b} \times \mathbf{c} = (-4, 16, -8)$, and $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = 52$.)

Q4 Suppose that $\mathbf{u}$ and $\mathbf{v}$ are unit vectors the angle between which is $\pi/4$. Let $\mathbf{a} = \mathbf{u} + 3\sqrt{2}\mathbf{v}$. By considering $\mathbf{a} \cdot \mathbf{a}$, find $|\mathbf{a}|$.

Q5 Let $\mathbf{u}$ and $\mathbf{v}$ be non-zero *parallel* vectors. Find $\mathbf{u} \times \mathbf{v}$.

Q6 Let $\mathbf{a}$ and $\mathbf{b}$ be non-zero vectors, simplify $(\mathbf{a} + 2\mathbf{b}) \times (3\mathbf{a} - 4\mathbf{b})$.

Q7 Let $\mathbf{a} = (-5, 4, 2)$ and $\mathbf{b} = (-2, 1, 2)$. Calculate $\mathbf{a} \times \mathbf{b}$ and hence find the two unit vectors perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

Q8 Let $\mathbf{a} = (1, 2, 3)$, $\mathbf{b} = (2, -1, 1)$ and $\mathbf{c} = (1, 4, -1)$.

a) Obtain the value of $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ by working out the value of a 3×3 determinant.

b) Without performing separate determinant expansions, write down the values of $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$ and $\mathbf{c} \cdot (\mathbf{b} \times \mathbf{a})$.