2A Multivariable Calculus 2013

Ex Sheet 6

Tutorial Exercises

T1 Sketch the wedge shaped region W (in the first octant) enclosed by the five planes $x = 0$, $y = 0$, $z = 0$, $x = 1$ and $y + z = 1$. Then evaluate

$$\iiint_W xy \, dxdydz.$$

T2 Sketch the solids whose volume is given by the following integrals

(a) $\displaystyle\int_0^2 dx \int_0^{3-3x/2} dy \int_0^{6-3x-2y} 1 \, dz$ (b) $\displaystyle\int_{-1}^1 dx \int_0^{1-x} dy \int_0^{1-x^2} 1 \, dz$

T3 A solid shell of variable density is in the form of the region lying between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$. The density ρ of the shell at the point (x, y, z) is given by $\rho(x, y, z) = \sqrt{x^2 + y^2 + z^2}$. Find the mass of the shell.

T4 Evaluate

$$\iiint z^2 \, dxdydz$$

throughout

a) the part of the sphere $x^2 + y^2 + z^2 = a^2$ $(a > 0)$ in the first octant,

b) the complete interior of the sphere $x^2 + y^2 + z^2 = a^2$ $(a > 0)$.

T5 Find grad f at the point P for

(a) $f = x^2 + y^2 - 3yz$, $P(1, 2, 1)$, (b) (b) $f = e^x \log(yz)$, $P(0, 2, 3)$,

(c) $f = \cos(yz) \log(xz)$, $P(1, 0, 3)$.

Further Exercises

F1 Evaluate

$$\iiint_T y \, dxdydz$$

throughout the tetrahedron T given by $x \geq 0, y \geq 0, z \geq 0, x + y + z \leq 1$.

F2 Use triple integtration to express the volume of the solid that is bounded by the given surfaces and evaluate the volume:

Lecture 11

- **Key Points:**
  - calculating triple integrals over regular domains
  - use triple integrals to calculate volumes and masses
  - formula for rewriting a triple integral in spherical coordinates
- **Read:**
  - Stewart Section 15.7 (p1041)
  - Stewart Section 15.9 (p1057)
- **Textbook Exercises:**
  - Exercises 15.7 (p1049) Qs 3–22, 39-42

Lecture 12

- **Key Points:**
  - evaluating integrals using spherical coordinates
  - definition and calculation of the gradient of a vector field
- **Read:**
  - Stewart Section 15.9 (p1057)
  - Stewart Section 14.6 (p960)
- **Textbook Exercises:**
  - Exercises 15.9 (p1061) Qs 1- 30

a) $z = x^2 + y^2 - 3,\ z = -x^2 - y^2 + 5,$

b) $y = x^2,\ z = -y + 4,\ z = 0.$

F3 Find the mass of the solid of constant density ρ that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z$, $z = 0$ and $z = 1$.

F4 Evaluate

$$\iiint_R \sqrt{x^2 + y^2 + z^2} e^{-(x^2+y^2+z^2)}\, dxdydz$$

where R is the interior of the sphere $x^2 + y^2 + z^2 = 1$.

F5 Let V be the interior of the sphere $x^2 + y^2 + z^2 = 1$. Without doing any integration, explain why

$$\iiint_V x^2\, dxdydz = \iiint_V y^2\, dxdydz = \iiint_V z^2\, dxdydz,$$

and why

$$\iiint_V z\, dxdydz \ = \ 0 \quad \text{and} \quad \iiint_V z^3\, dxdydz = 0.$$

F6 Evaluate

$$\iiint_R \frac{z}{\sqrt{x^2 + y^2 + z^2}}\, dxdydz$$

where R is the interior of the sphere $x^2 + y^2 + z^2 = 2z$.

F7 Evaluate

$$\iint_R \frac{1}{(x^2 + y^2 + z^2)^2}\, dxdydz$$

where R is the region in the first octant *outside* the sphere $x^2 + y^2 + z^2 = 1$.

F8 Find grad f for

(a) $f = x \sin(y)$, (b) (b) $f = x \log(x + 3z)$, (c) $f = \sqrt{zy} \cot(x + y)$.

Harder challenge problems[1]

[1] Only attempt these if you have been able to do all the other problems successfully.

F9 Find the volume of the region lying inside the cylinder $x^2 + 4y^2 = 4$ above the xy-plane, and below the plane $2 + x$.

F10 Find $\iiint_R z\, dV$, over the region R satisfying $x^2 + y^2 \le z \le \sqrt{2 - x^2 - y^2}$.