Handout - Lect 1 -

COBWEBBING

We can then investigate the behaviour by the method of *cobwebbing*. The idea is as follows.

- Choose a starting value N_0, and begin at the point (N_0, N_0) in the (N_t, N_{t+1})-plane.
- Draw a vertical line to the curve $N_{t+1} = f(N_t)$; this reaches the curve at the point $(N_0, f(N_0)) = (N_0, N_1)$.
- Draw a horizontal line to the diagonal $N_{t+1} = N_t$; this reaches the diagonal at the point (N_1, N_1).
- Repeat the process to arrive at (N_2, N_2), and then indefinitely until the behaviour of the equation with this starting value becomes clear.
- If necessary, do the same with other starting values.

Graphical example

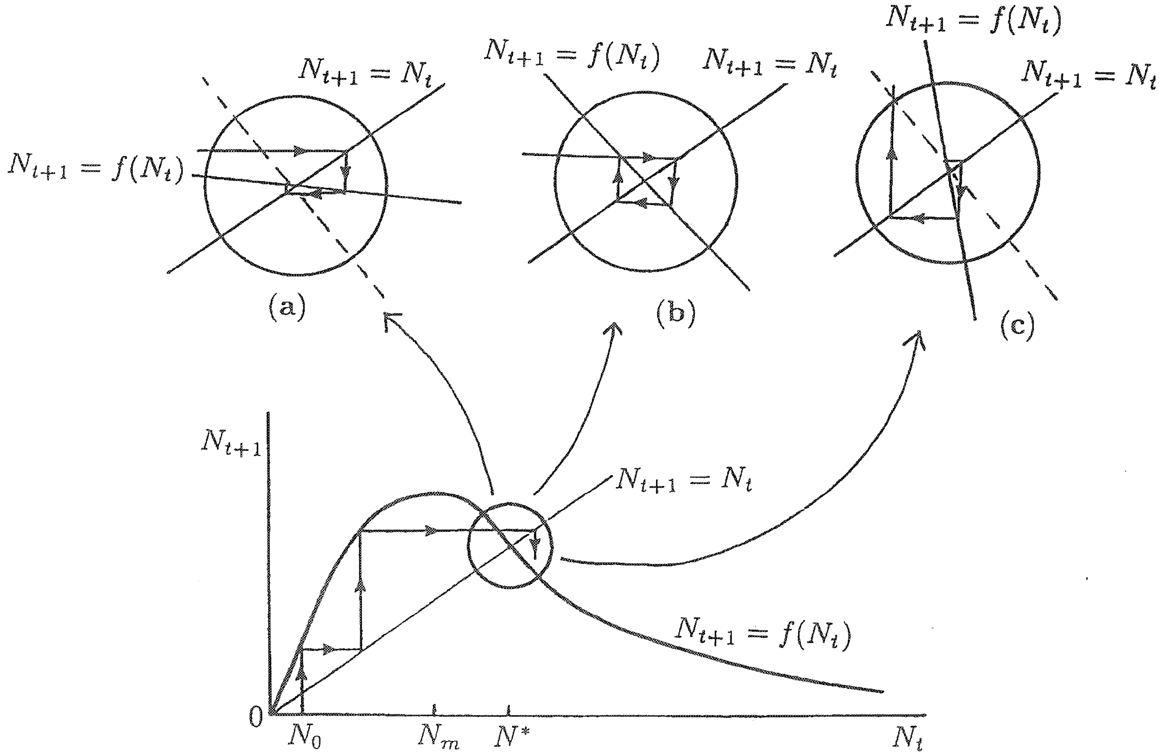

Fig. 2.3a-c. Local behaviour of N_t near a steady state where $f'(N^*) < 0$. The enlargements show the cases where: (a) $-1 < f'(N^*) < 0$, N^* is stable with decreasing oscillations in any small perturbation from the steady state. (b) $f'(N^*) = -1$, N^* is neutrally stable. (c) $f'(N^*) < -1$, N^* is unstable with growing oscillations.

Time behaviour

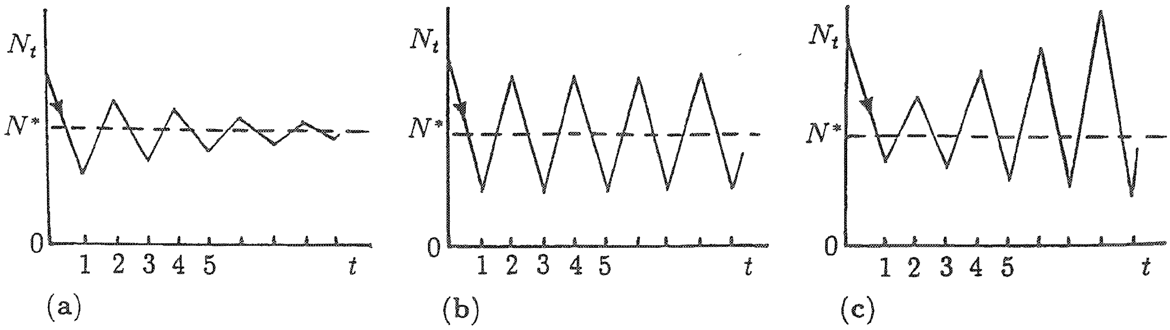

Fig. 2.4a-c. Local behaviour of small perturbations about the equilibrium population N^* with (a), (b) and (c) corresponding to the situations illustrated in Fig. 2.3 (a), (b) and (c) respectively: (a) is the stable case and (c) the unstable case.

Illustration of Chaos

The solution is globally bounded, but does not tend to a steady state. The solution in time seems to wander about in a random way.

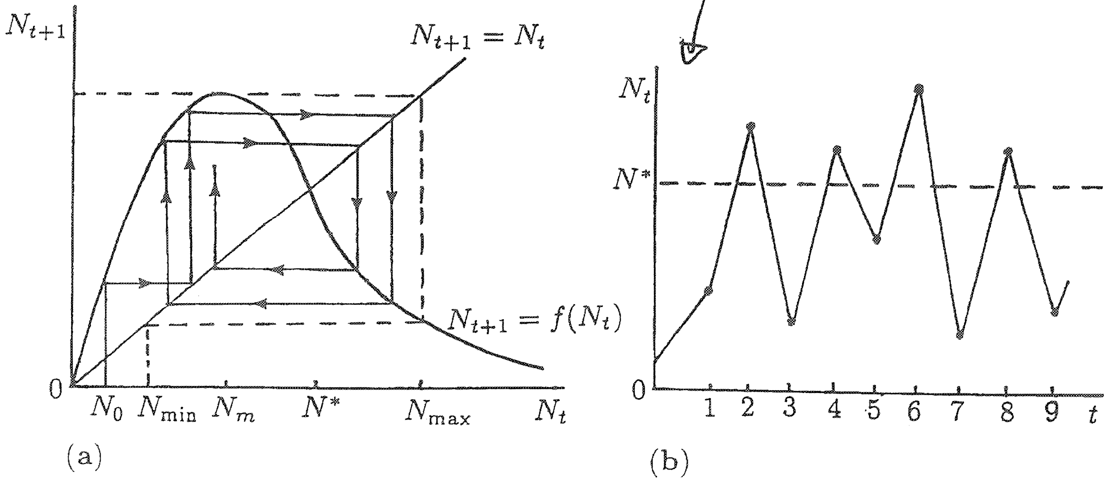

Fig. 2.5a,b. (a) Cobweb for $N_{t+1} = f(N_t)$ where the eigenvalue $\lambda = f'(N^*) < -1$. (b) The corresponding population behaviour as a function of time.