2A Multivariable Calculus 2013

Ex Sheet **4**

Tutorial Exercises

T1 By reversing the order of integration, evaluate

$$\text{(a)} \int_0^1 dy \int_y^1 \sinh(x^2)\,dx, \quad \text{(b)} \int_1^e dx \int_{\log x}^1 \frac{e^{-y^2}}{x}\,dy.$$

T2 Find the volume of the prism whose base is the triangle with vertices at $(0,0,0)$, $(1,0,0)$ and $(0,2,0)$, which has sides parallel to the z-axis and the top of which is the plane $3x+2y+z=10$.

T3 Evaluate

$$\int\int xy^2\,dx\,dy$$

over the region in the first quadrant that lies outside the circle $x^2+y^2=1$ but inside the circle $x^2+y^2=9$.

T4 Evaluate

$$\int\int_R y(x^2+y^2)\,dx\,dy$$

where R is

a) the part of the interior of the circle $x^2+y^2=2x$ that lies in the first quadrant,

b) the part of the interior of the circle $x^2+y^2=2x$ that lies above the line $y=x$.

c) the region in the first quadrant inside $x^2+y^2=4ax$ but outside $x^2+y^2=2ax$, where $a>O$.

T5 Evaluate the following integrals by converting to polar coordinates

$$\text{(a)} \int_0^1 dy \int_y^{\sqrt{2-y^2}} 3(x+y)\,dx, \quad \text{(b)} \int_0^2 dx \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy.$$

Further Exercises

F1 By changing the order of integration, evaluate the following integrals

$$\text{(a)} \int_0^1 dx \int_x^1 \frac{x}{1+y^3}\,dy, \quad \text{(b)} \int_0^1 dx \int_{x^2}^1 x^3\sqrt{y^3+15}\,dy,$$

$$\text{(c)} \int_0^2 dx \int_{x^3}^8 \frac{x^2}{(1+y^2)^2}\,dy.$$

Lecture 7

- **Key Points:**
  - changing the order of integration
  - using double integrals to calculate volumes
  - expressing an integral in polar coordinates
- **Read:**
  - Stewart Section 15.3 (p1012)
  - Stewart Section 15.4 (p1021)
- **Textbook Exercises:**
  - Exercises 15.3 (p1020) Qs 43–56

Lecture 8

- **Key Points:**
  - evaluating integrals using polar coordinates
  - calculating volumes using polar coordinates
  - formulating an appropriate integral from a word problem
- **Read:**
  - Stewart Section 15.4 (p1021)
- **Textbook Exercises:**
  - Exercises 15.4 (p1026) Qs 1–34

F2 Find the volume of the section of the cylinder $x^2 + y^2 = 1$, between the planes $z = 0$ and $x + y + z = 2$.

F3 Use polar coordinates to evaluate

$$\iint_D \cos\left(x^2 + y^2\right) \, dA$$

where D is the region in the first quadrant between the circles with centre $(0, 0)$ and radii 1 and 3 respectively.

F4 Evaluate

$$\iint_D \sqrt{x^2 + y^2} \, dA$$

where D is the disk with centre $(0, 1)$ and radius 1.

F5 Evaluate

$$\int\int \frac{y^2}{x^2 + y^2} \, dx \, dy$$

over the region in the first quadrant that lies inside the circle $x^2 + y^2 = a^2$, where $a > 0$. What is the value of the same integral over the entire disc enclosed by this circle?

F6 Evaluate

$$\int\int x\sqrt{x^2 + y^2} \, dx \, dy$$

over the finite region in the first quadrant enclosed by the x-axis, the line $y = \sqrt{3}x$ and the circle $x^2 + y^2 = a^2$, where $a > O$.

F7 An inflatable rubber tent takes the form of the paraboloid $z = 1 - x^2 - y^2$ for $z \geq 0$. Find the volume of air which it encloses.

F8 A dummy funnel on a passenger steamer is to be used as a water tank. The tank is to have vertical sides, a horizontal base and slanting plane top. Find the volume of the tank if the base is the plane $z = 0$, the top is the plane $x + 3z = 24$ and the sides are determined by the circular cylinder $x^2 + y^2 = 9$.

[1] Harder challenge problems

[1] Only attempt these if you have been able to do all the other problems successfully.

F9 Evaluate the integral

$$\int_0^1 dy \int_{\sin^{-1} y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x} \, dx.$$

F10 (a) A cylindrical drill with radius r_1 is used to bore a hole through the center of a sphere of radius r_2. Find the volume of the ring shaped solid that remains.
(b) Express the volume in part (a) in terms of the height h of the ring. Notice that the volume depends only on h not on r_1 or r_2.