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Large- to small-scale dynamo in domains of large aspect ratio: Kinematic Regime

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ABSTRACT

The Sun's magnetic field exhibits coherence in space and time on much larger scales than the turbulent convection that ultimately powers the dynamo. In this work, we look for numerical evidence of a large-scale magnetic field as the magnetic Reynolds number, R_m, is increased. The investigation is based on the simulations of the induction equation in elongated periodic boxes. The imposed flows considered are the standard ABC flow with wavenumber $k^u = 1$ (small-scale) and a modulated ABC flow with wavenumbers $k^u = m, 1, 1 \pm m$, where m is the wavenumber corresponding to the long-wavelength perturbation on the scale of the box. The critical magnetic Reynolds number $R_m^{\rm crit}$ decreases as the permitted scale separation in the system increases, such that $R_m^{\rm crit} \propto [L_x/L_z]^{-1/2}$. The results show that the α-effect derived from the mean-field theory ansatz is valid for a small range of R_m after which small scale dynamo instability occurs and the mean-field approximation is no longer valid. The transition from large to small-scale dynamo is smooth and takes place in two stages: a fast transition into a predominantly small-scale magnetic energy state and a slower transition into even smaller scales. In the range of R_m considered, the most energetic Fourier component corresponding to the structure in the long x-direction has twice the lengthscale of the forcing scale. The long wavelength perturbation imposed on the ABC flow in the modulated case is not preserved in the eigenmodes of the magnetic field.

Key words: Sun: magnetic fields | methods: numerical | (magnetohydrodynamics) MHD | dynamo

1 INTRODUCTION

The evolution and sustainment of the solar magnetic field are believed to be governed by a dynamo process. In particular, small-scale fields that are spatially coherent at scales similar to or smaller than those of the turbulent velocity fields, are known to be produced by the small-scale dynamo (Meneguzzi et al. 1981). It is much more difficult to explain the generation of large-scale magnetic fields that exhibit coherence in space and time on much larger scales than the turbulent convection that ultimately powers the dynamo.

Traditionally such large-scale magnetic fields are modelled under the mean-field theory ansatz, which is valid under the crucial assumption of a separation of scales between the scale of the system (for example, the depth of the solar convection zone) and the dominant scales of the flow (Moffatt 1978; Krause & Rädler 1980). It enables the consideration of large-scale fields, $\bar{\mathbf{B}}$, separate from the small-scale fluctuations, $\mathbf{u}$, $\mathbf{b}$. The theory outlines how the small-scale turbulent motions produce a coherent global field through the interaction with small-scale magnetic fields. In the basic case, where the underlying velocity is isotropic and homogeneous, the induction equation that describes the evolution of a large-scale magnetic field depends on two quantities: α, the mean induction, and β, the turbulent diffusivity. Then the amplification of large-scale magnetic fields occurs due to the so-called $\alpha-$effect, $\alpha\bar{\mathbf{B}}$, which is related to the mean electromotive force, $\langle \mathbf{u} \times \mathbf{b} \rangle$, where $\langle . \rangle$ is a spatial average with respect to the small length scale (Moffatt 1978).

In the limit, where the magnetic Reynolds number (which describes the ratio between advective and diffusive processes), R_m, is small, the coefficients can be calculated under the first order smoothing approximation (FOSA). However, if R_m is large, the fluctuating small-scale magnetic field grows exponentially faster than the large scales, thus disregarding the scale separation assumption (Cattaneo & Hughes 2009). Many authors have questioned the use of mean-field theory ansatz with respect to the solar dynamo (Galloway & Frisch 1986; Boldyrev et al.

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2005; Courvoisier et al. 2006; Cattaneo & Hughes 2009; Cattaneo & Tobias 2014; Cameron & Alexakis 2016).

In this paper, we investigate the validity of the $\alpha-$effect approximation and its dependence on the separation of scales in the system. We study numerically the kinematic dynamo induced by highly helical single- and multi-scale flows in elongated periodic boxes as a simplified model to understand the dynamo action in astronomical bodies. We choose to work with periodic cuboid domains, because it allows us to study the dynamo properties of spatially periodic cellular flows. We vary the scale separation in the system through the reconstruction of the computational domain and the characteristic lengthscale of the imposed velocity field.

The first flow under consideration is the well-known ABC flow (named after Arnold, Beltrami & Childress), recently reviewed by Galloway (2012). The second is a modulated version of the first, with imposed long-wavelength perturbation on the scale of the system. Such periodic flows are guaranteed to produce dynamo action for a wide range of finite values of R_m (Roberts 1970). We study the kinematic dynamo action in domains of various aspect ratios, to examine the effect of changing the scale separation of the system as well as the effects of imposing a two-scale perturbed flow on the system in the kinematic regime. The first part (ABC flow) of the problem considered here has recently been investigated in a different way by Cameron & Alexakis (2016) who carried out a theoretical study of kinematic dynamo using the Floquet formulation that was originally devised for the Mathieu equation (Whittaker & Watson 1902). The authors adopted the concepts in (Roberts 1970) to solve the induction equation in the presence of the ABC flow for arbitrary large scale separations, showing a transition from a large- to small-scale dynamo through a change in the growth rate. The results from this analysis are in general agreement with the findings presented below as the scale separation becomes arbitrarily large. However, their paper does not include the results for Floquet waves with lengthscales comparable to the size of the imposed flow, although Galanti et al. (1992) noted such eigenmodes play a crucial role in maintaining the dynamo for large R_m. In this paper we utilise a discrete wavenumber decomposition that is determined by the aspect ratio of the computational domain. This allows us to investigate the transition from a large- to small-scale dynamo with a special emphasis on the lengthscale selection of the fastest growing eigenmode of magnetic field and focus on the comparison of the results from the classic ABC and the modulated perturbed flow cases.

Our study is based on the results of numerical simulations using a pseudospectral method in a triply-periodic domain and analytic estimates based on the scale separation arguments. The choice of the imposed flows allows for the scenario wherein a large-scale dynamo would be obtained at the onset. We then vary magnetic diffusivity in order to study how the dynamo mechanism changes as we increase R_m. There are two possibilities: (i) the large-scale structure is preserved due to either the large aspect ratio of the system or the imposed long-wavelength modulation; or (ii) the energy is passed from the large scales to the small scales as the system becomes less diffusive. In the second case, the transition can either be a discontinuous jump between different modes for the different types of dynamo, or the transition is continuous and the two processes morph into one another. A similar study by Ponty & Plunian (2011) was performed for the Roberts flow geometry, demonstrating that the scale of the turbulence has an insignificant effect on the large-scale dynamo at the onset, and yet the further increase of R_m can give rise to a small-scale dynamo. In this work we determine whether large-scale modes of the magnetic field persist in elongated boxes as we increase R_m in the kinematic regime, in the presence of the small-scale ABC flow as well as the modulated ABC flow with imposed long-wavelength perturbation. The extension into the nonlinear regime will follow in subsequent work (Shumaylova et al. 2017).

2 NUMERICAL SET UP

2.1 Governing Equations

We consider incompressible magnetohydrodynamics (MHD) to describe the evolution of magnetic field in the presence of the prescribed flows, see section 2.3. The induction equation describes the growth or decay of the magnetic field, $\mathbf{B}$, as it interacts with the velocity field, $\mathbf{u}$, in the kinematic regime,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + R_m^{-1} \nabla^2 \mathbf{B}, \qquad (1)$$
$$\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{u} = 0,$$

where $R_m = U(\eta k^u)^{-1}$ is the magnetic Reynolds number based on the small velocity scale, $(k^u)^{-1}$, and magnetic diffusivity, η.

2.2 Numerical Code

The equations are solved with triply-periodic boundary conditions using the Snoopy code, described in Lesur & Longaretti (2007), which utilises a spectral method. In order to observe the growth of the mean magnetic field in this model the size of the domain must be significantly larger than the scale of the turbulent motions, $k^u = 1$, that corresponds to the shortest side of 2π in the computational domain. However, this then requires simulations to be run in computationally expensive large boxes for very long durations. Yousef et al. (2008) alleviated this problem somewhat by using boxes with a large aspect ratio so that one direction of the domain is much longer than the others: $L_x \gg L_y, L_z$. It is sufficient to run with a spatial resolution of 32 to 64 points on 2π depending on R_m. In this paper we consider dynamo action in the following periodic domains and the wavenumber corresponding to the largest scale in the box:

$$L_x = [32, 20, 16, 10, 8] \times 2\pi \qquad k_x^{\min} = 2\pi/L_x$$
$$L_y = 2 \qquad \times 2\pi \qquad k_y^{\min} = 0.5$$
$$L_z = 1 \qquad \times 2\pi \qquad k_z^{\min} = 1.$$

The choice of the aspect ratios is based on the idea of allowing different wavelength modes to grow in each direction to study the magnetic field eigenmode scale selection. The prescribed aspect ratio of the non-preferred shorter directions is selected on the basis of Galanti et al. (1992). Their results for $R_m \leq 100$ indicate the favouring of scales of the magnetic field that are no more than twice the characteristic scale of the imposed field. However, we find that the flow has no impact on the structure of magnetic field in non-preferred directions in the mean-field limit and thus, there is no need to run full 3D calculations in this limit. We recognise that, nonetheless, the domain restriction may become an issue for much larger R_m .

2.3 Prescribed Flow

We investigate the spatial scale selection for coherent structures of magnetic field produced by dynamo action in the presence of a velocity field that has a single spatial scale (classic ABC flow) and a two-scale velocity field with a long wavelength modulation. The two prescribed flows are made up of small structures on scale of 2π.

Firstly, we investigate the dynamo action associated with the well-known ABC flow:

$$\mathbf{u}_{\rm ABC} = \begin{bmatrix} A\sin(k^u z) + C\cos(k^u y) \\ B\sin(k^u x) + A\cos(k^u z) \\ C\sin(k^u y) + B\cos(k^u x) \end{bmatrix}$$

The flow is considered to be a prime candidate for the amplification of the magnetic field in periodic domains due to its "twist" property of maximal helicity. Many authors have studied the properties of the ABC flows (Arnold & Korkina 1983; Galloway & Frisch 1984; Dombre et al. 1986; Galloway & O'Brian 1994; Archontis et al. 2002), including in more recent studies (Alexakis 2011; Galloway 2012; Bouya & Dormy 2013; Sadek et al. 2015; Cameron & Alexakis 2016). In this paper we are considering the most studied, classic case, $A = B = C = 1$ with the forcing wavenumber $k^u = 1$ that has the largest number of symmetries.

Secondly, we investigate the dynamo action associated with a modulated ABC flow. We impose a long wavelength perturbation (comparable to the size of the box) on a set of replicated classical ABC cells, $\mathbf{u}_m = \nabla \times (\cos(mx)\mathbf{u}_{ABC})$, where $m = 2\pi/L_x$ is the wavenumber that corresponds to the longest scale in x,

$$\mathbf{u}_m = Dk^u\cos(mx) \begin{bmatrix} A\sin(k^u z) + C\cos(k^u y) \\ B\sin(k^u x) + A\cos(k^u z) \\ C\sin(k^u y) + B\cos(k^u x) \end{bmatrix}$$

$$+Dm\sin(mx) \begin{bmatrix} 0 \\ B\cos(k^u x) + C\sin(k^u y) \\ -A\cos(k^u z) - B\sin(k^u x) \end{bmatrix}$$

The flow is strongly helical and able to maintain dynamo action for a wide range of R_m. In this paper we are considering the classic setup, $A = B = C = D = 1$, with the forcing wavenumber $k^u = 1$ and varying $m = 2\pi/L_x$. Thus, we examine the effects of the imposed flow with wavenumbers $m, 1, 1 \pm m$ on the evolution of magnetic field. Both flows are constructed by replicating the $[2\pi]^3$ cubes in the directions that have an aspect ratio greater than one, i.e. x and y, (see Figure 1).

3 RESULTS

Under the framework of kinematic mean-field theory that addresses the growth of a weak seed field and predicts the growth rate and structure of the generated magnetic field, we solve the induction equation for the mean magnetic field.

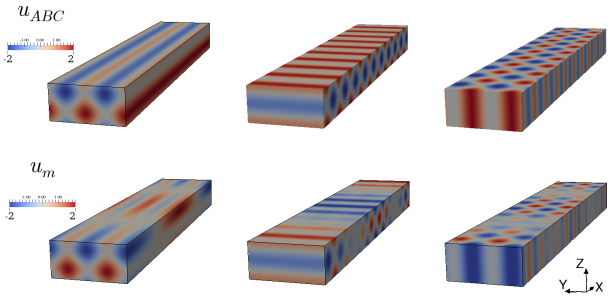

Figure 1. The velocity field components for the classic ABC (top row) and modulated (bottom row) flows in a box of aspect ratio $[10, 2, 1]$. The magnitude of the body force has been chosen such that the velocities A, B, C are all unity in code units $[LT^{-1}]$ and D is in units $[L]$, and the numerical resolution was 32 collocation points on $L = 1$. The modulation of $m = 1/10[L^{-1}]$ produces a visible difference from a regular ABC flow. The values range between $\pm 2L/T$, with red values being positive and blue values negative.

The mean field is amplified by the $\alpha-$effect described in section 1. We derive the α coefficients under the assumption of a separation of scales and $R_m \ll 1$ by averaging over small scales:

$$\alpha_{\rm ABC} = \frac{A^2}{\eta}k^u \quad (2)$$

$$\alpha_m = \frac{A^2 D^2}{\eta}k^u\cos^2(mx) \quad (3)$$

The derived α coefficients can be used to solve the mean-field induction equation as an eigenvalue problem, in order to find the onset of dynamo action for the fastest growing mode, its growth rate and spatial structure. Moffatt (1978) noted that the isotropic α-coefficients like $\alpha_{\rm ABC}$ that are constant in space, provide an estimate for the growth rate for the solution of the form $\exp(ikx + \sigma t)$:

$$\sigma = \alpha_{\rm ABC}k - \eta k^2.$$

The spatially dependent $\alpha-$effect for the modulated flow yields two solutions: odd- and even-wavenumber eigenmodes that grow with different exponential rates. We compare the asymptotic approximation with the results from the simulation runs for boxes of various aspect ratios.

Simulations are initialised with a random, zero-mean, weak seed magnetic field of amplitude 10^{-5} and the prescribed velocity fields in section 2.3. The magnetic Reynolds number is defined according to Galanti et al. (1992),

$$R_m = \frac{\langle \mathbf{u}\cdot\mathbf{u}\rangle}{\sqrt{3}\langle \mathbf{w}\cdot\mathbf{w}\rangle^{1/2}}\eta^{-1}, \quad (4)$$

$$\mathbf{w} = \nabla \times \mathbf{u},$$

such that $R_m = \eta^{-1}$ for the classic ABC flow. In the kinematic regime, this is the critical parameter that determines the onset of dynamo action. $R_m^{\rm crit}$ is the threshold for the amplification of the magnetic field, i.e. $\sigma(R_m^{\rm crit}) = 0$. The onset can be predicted analytically under the mean-field theory framework that relies on the scale separation in the system. Numerically, $R_m^{\rm crit}$ is determined using linear interpolation between the values of R_m, corresponding to the decay of the magnetic field and the slowest growing dynamo. The results for the critical magnetic Reynolds number as a function of

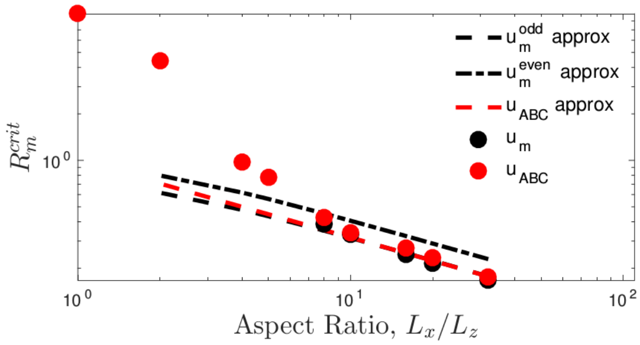

Figure 2. $R_m^{\rm crit}$ as a function of domain aspect ratio in elongated periodic boxes in the presence of a single-scale flow, $\mathbf{u}_{\rm ABC}$, (red) and a two-scale flow, $\mathbf{u}_m$ (black). The filled circle corresponds to the value of $R_m^{\rm crit}$ obtained from direct numerical simulations. The dashed lines show the mean field approximation based on the α-effect mechanism. Two black dashed lines refer to two different eigenmodes (odd-wavenumber solution onsets at an earlier R_m in comparison to the even one).

aspect ratio, L_x/L_z, are shown in Figure 2. The analytical approximation is shown with dashed lines. We find that the onset of dynamo action in the kinematic regime occurs at lower values of R_m depending on the aspect ratio of the periodic domain, scaling as

$$R_m^{\rm crit} \propto (L_x/L_z)^{-1/2}, \qquad \text{where } L_z \sim 1/k^u \tag{5}$$

for large enough aspect ratio ($L_x/L_z > 8$), which allows large-wavelength modes to grow in the system. Hence, the mean-field approximation given by the dashed lines in Figure 2 provides a fairly good estimate for the onset of dynamo action, as expected, since the ansatz is only valid in the limit of large-scale separation.

Figure 3 presents the evolution of the maximum growth rate of the magnetic field as a function of R_m in domains of various aspect ratios. Here we calculate the growth rate, σ, of $B_{\rm rms} = \langle B^2 \rangle^{1/2}$ over the total duration of the linear regime. Each point in the figure corresponds to a three-dimensional simulation. The approximation curves are determined from the eigenvalue problem based on the derived $\alpha-$effect coefficients. The curves and the results from the numerical runs show a smooth transition from the large-scale fastest growing mode at $R_m \ll 1$ to dominant modes that are smaller than the size of the box itself but longer than the characteristic velocity scale. What the figure does not show is that the no-longer-dominant large-scale modes continue to grow in the system with smaller growth rates than the shorter dominant modes. We also found that in the elongated domains, we do not reproduce the classic 'two-window' cube result of the classic ABC dynamo with $k^u = 1$ (Galloway & Frisch 1984).

The transition from large- to small-scale dynamo can also be seen in Figure 4. We measure the lengthscale of the magnetic field using two scales,

$$l_{\bar{B}} = \max_i \left[\frac{\langle (\partial \bar{B}_i/\partial x)^2 \rangle_x}{\langle (\bar{B}_i)^2 \rangle_x} \right]^{-1/2}, \tag{6}$$

$$l_B = \left[\frac{\langle (\nabla \times \mathbf{B})^2 \rangle}{\langle \mathbf{B}^2 \rangle} \right]^{-1/2}, \tag{7}$$

where eq. (6) is the measure of the characteristic lengthscale

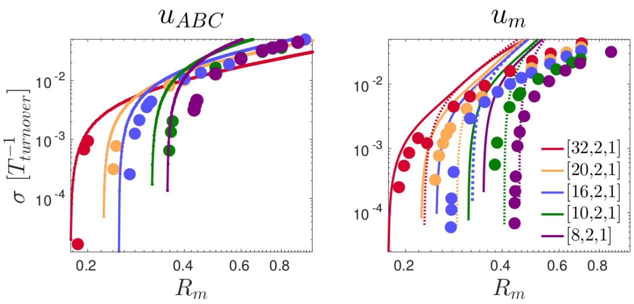

Figure 3. Growth rate of $B_{\rm rms}, \sigma$, versus magnetic Reynolds number in the presence of $\mathbf{u}_{\rm ABC}$ (left) and $\mathbf{u}_m$ (right). Asymptotic approximation for σ is given by the solid and dashed (modulated case yields odd and even mode solutions) lines. Solid circles correspond to the growth rates from direct numerical simulations.

of the mean field (calculated by isolating large-scale dependence on x by low-pass filtering in Fourier space) as proposed by Yousef et al. (2008). Eq. (7) is the measure of the kinematic dissipation lengthscale as defined in Oruba & Dormy (2014). Both scales are measured at the final state of the simulation at the end of the period of exponential growth of the magnetic field. The derivatives are calculated in Fourier space. The values of $l_{\bar{B}}$ and l_B are plotted against R_m in Figure 4. The scale separation leads to a remarkable difference in the structure of the mean magnetic field at low R_m, i.e. the field grows on the scale of the box. As we increase R_m, both the mean-field and the dissipation scales decrease and are roughly matched by the scaling $\propto R_m^{-1/2}$. The dominance of the large-scale magnetic field is replaced by the fast growing small scales as the magnetic Reynolds number increases. This is not a surprising result for the classic ABC flow that is known to generate structures comparable in size to the flow itself at R_m outside the mean-field limit (Galanti et al. 1992). However, the result is unexpected in the modulated case: in the kinematic regime the presence of a long wavelength perturbation in the prescribed flow ($\mathbf{u}_m$) is not preserved in the fastest growing eigenmodes of the magnetic field.

In Figure 4 we can distinguish two separate stages of the transition from large- to small-scale dynamo, more clearly shown in Figure 5 using the magnetic field dissipation scale. We find that the measure of the magnetic field given in eq. (7) scales as $\sim R_m^{-1}$ at low R_m when magnetic energy is predominantly in large scales ($k < k^u/2$). This result can be obtained by considering the approximation for the fluctuating magnetic field in terms of the mean-field, $\mathbf{b} \sim R_m \hat{\mathbf{B}}$ and the scaling law for the critical $R_m^{\rm crit}$ in (5) that can be used to show that $l_B \propto R_m^{-1}$ in the mean-field limit. For $R_m > O(1)$, the rate of scaling with R_m goes down to $\sim R_m^{-0.5}$ in both cases. This observation is especially apparent in the boxes of larger aspect ratios. This result is assumed to be due to the effect of the boundary layers of thickness $R_m^{-1/2}$ forming in the domain for large R_m (Proctor & Weiss 2014).

Therefore, we may conclude that the dynamo scale in the kinematic regime primarily depends on the magnetic Reynolds number preassigned to the system as a parameter. The long wavelength perturbation imposed on the ABC flow in the modulated case is not preserved in the eigenmodes of

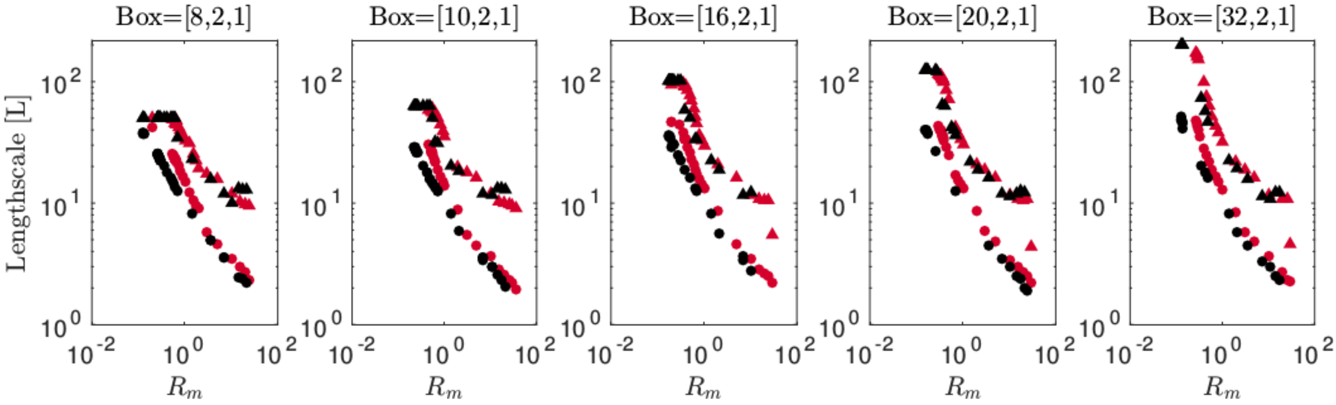

Figure 4. Measure of the kinematic lengthscale in eq. (7) (filled circle) and mean-field in eq.(6) (filled triangle) from direct numerical simulations of kinematic dynamo in the presence of $\mathbf{u}_{\mathrm{ABC}}$ (red) and $\mathbf{u}_m$ (black).

the magnetic field and large-scale structures are diminished as we leave the mean-field limit.

Archontis et al. (2002) noticed that there are two stages to the kinematic regime: in the first stage the magnetic field evolves according to the symmetry of the imposed flow, and in the second stage symmetry breaking occurs, i.e. the topology of the magnetic field changes and the field grows exponentially while maintaining this topology. In Figure 6 we plot the scale of the structure in the long x-direction, k_x^{-1}, that has the most energy at every time step of the second stage of the kinematic regime (note that it does not necessarily correspond to the peak of the energy spectrum). The lengthscale, k_x^{-1}, is equal to the size of the box at the onset of dynamo action in boxes of all aspect ratios. However, as we increase R_m, the large-scale field is replaced by smaller structures that are still much greater than the forcing lengthscale, $k^{u-1} = 1$. Interestingly, in the considered interval of $R_m \in (1, 40)$,

$$k_x \to \frac{k^u}{2} \pm m \qquad \text{as } R_m \text{ is increased,}$$

where $m = 2\pi/L_x$ is the wavenumber corresponding to the longest scale on the box. That is, the fastest growing eigenmode of the magnetic field has a spatial period roughly twice that of the 'forcing' flow ($k^u = 1$). This cannot currently be compared with the results of (Cameron & Alexakis 2016) since they do not consider the possibility that $k_x = 0.5$. We can draw a parallel between this result and the experimental observation first made by Faraday (1831), in which he noted that surface waves in a fluid-filled cylinder under vertical excitations were generated with twice the period of the excitation itself. And indeed, the first detailed theory relevant to the study of a periodically time-varying system was presented by Mathieu (1868), in which the problem of the vibration of an elliptical membrane was analysed. A similar phenomenon of unstable modes 'choosing' lengthscales that are twice that of the imposed forcing has been observed in many branches of physics and engineering. This occurring pattern is preferred by certain restricted geometries, including the one considered here and Galanti et al. (1994). For a less constrained geometry, the preferred pattern may vary.

CONCLUSIONS

In this paper we have studied kinematic dynamo action in elongated triply-periodic cuboids as a numerical solution of the induction equation with two prescribed velocity fields. The imposed flows are the classic $1:1:1$ ABC flow and the ABC flow modulated with a long-wavelength perturbation. Both flows were strongly helical, which ensured that a large-scale dynamo was operating at the onset of dynamo action. The properties of these dynamos can be adequately modelled by a mean field approximation, but only for $R_m \lesssim 1$ near the onset. At large R_m the assumption of scale separation in the system becomes meaningless, since small-scale modes grow exponentially faster than the large-scale modes that were dominant at low R_m, even in the elongated domains that favour large scale structures and even for the modulated flow that contains a large-scale component. Therefore, we were able to study the transition from a large-scale dynamo at the onset to a small-scale dynamo by increasing the magnetic Reynolds number for both the ABC flow (single small-scale) and modulated flow (two-scale).

At the onset, the fastest growing eigenmode is the mean-field mode $(2\pi/L_x, 0, 0)$ due to the large aspect ratio of the computational domain. As the magnetic Reynolds number is increased, small-scale dynamo action becomes possible, increasing the overall growth rate of the dynamo. We find that the transition between the two is continuous, i.e. the large-scale dynamo is present even when the small-scale dynamo is dominant in the system. The initial rate of the transition varies slightly for the imposed flows, with the modulated flow exciting small-scale dynamo action sooner than the classic ABC flow. The second stage of the transition from large- to small scales that is initiated when most magnetic energy is in smaller scales ($k > k^u/2$), shows a significant decrease in the scaling rate with R_m for both flows. The critical magnetic Reynolds number that corresponds to the onset of dynamo action scales with the aspect ratio of the domain. This result could be of interest for groups working on experimental dynamos.

Our future work will consider the nonlinear regime, that is more applicable to the astrophysical context. Here, the fluid flow $\mathbf{u}$ is no longer prescribed, but the influence of $\mathbf{B}$ on $\mathbf{u}$ via the Lorentz force is included. The preliminary results from the study of dynamic dynamos (in the presence

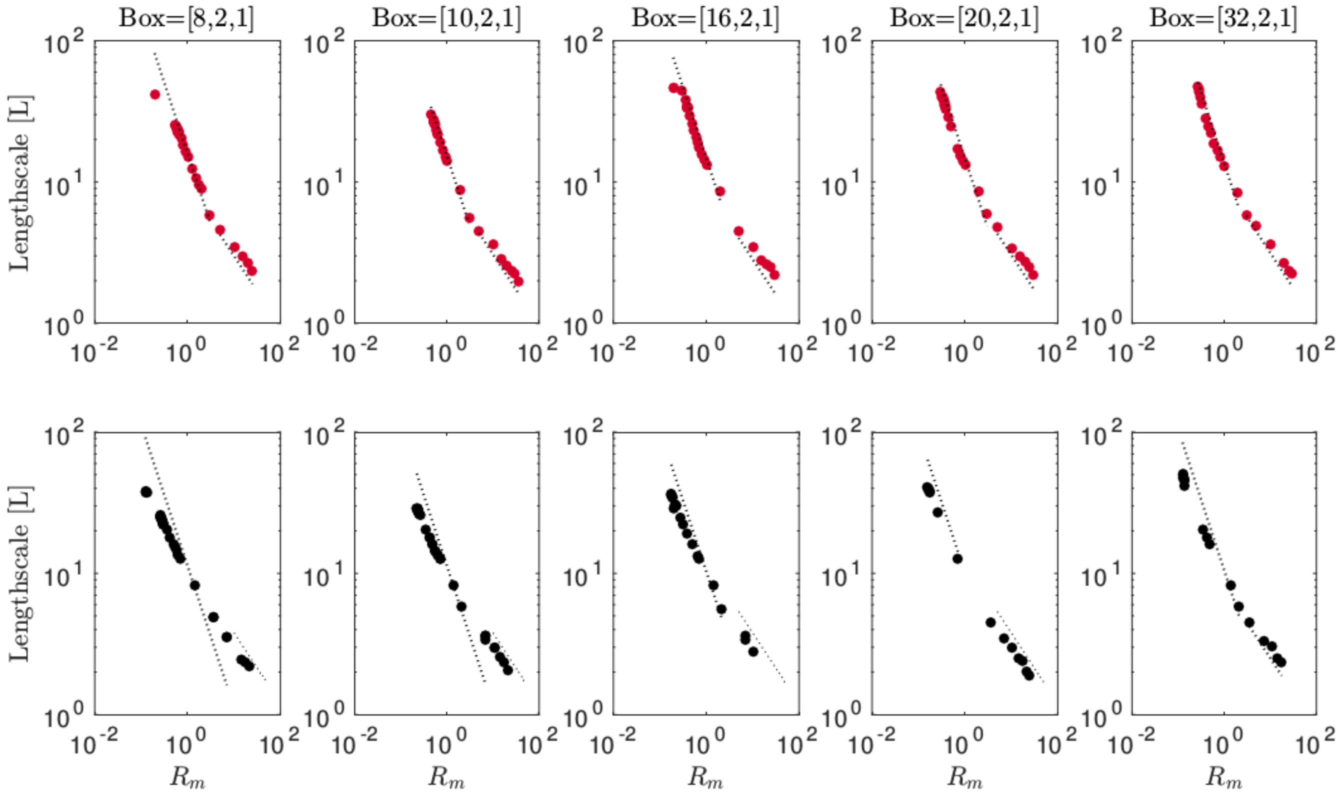

Figure 5. Measure of the kinematic lengthscale in eq. (7) vs R_m. The dotted line shows the approximation curve as a fuction of R_m in the presence of $\mathbf{u}_{\mathrm{ABC}}$ (red) and $\mathbf{u}_m$ (black) for two transition stages roughly defined by the turning point where magnetic energy is predominately in smaller scales $k > k^u/2$. During the first stage, the lengthscale decreases with R_m^{-1} and $R_m^{-0.5}$ during the second one.

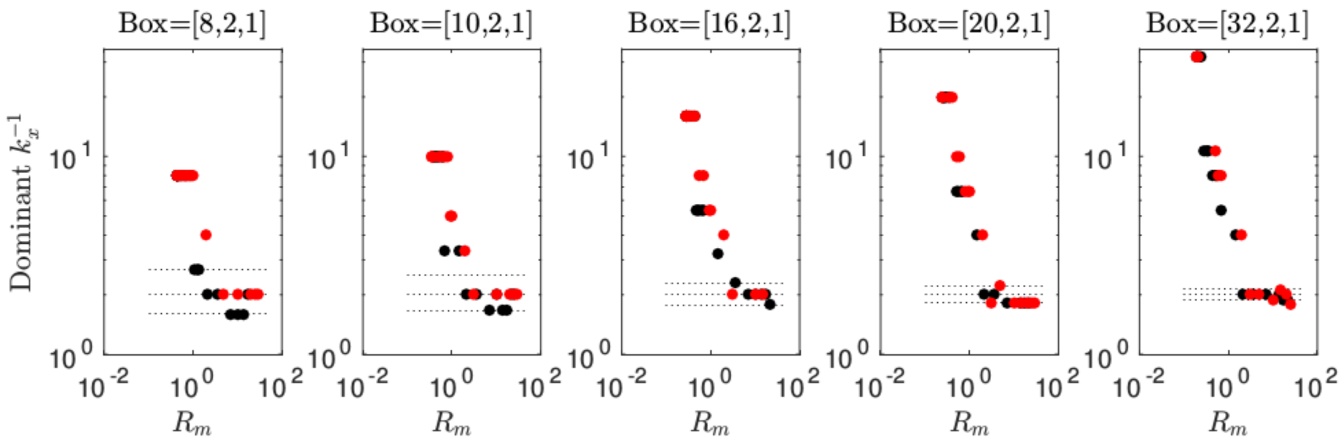

Figure 6. Measure of scale of the structure in x-direction, k_x^{-1}, vs R_m in the presence of $\mathbf{u}_{\mathrm{ABC}}$ (red) and $\mathbf{u}_m$ (black). The dotted line shows $L = 2, 2 \pm 2\pi/m$, where $m = 2\pi/L_x$ is the wavenumber corresponding to the longest scale on the box.

of viscous forcing related to the velocity fields discussed in the current paper) show that large-scale dynamo action can be found in the nonlinear regime, whereas the same forcing function in the kinematic regime produced a small-scale dynamo; it will therefore be interesting to see whether further analysis of the parameter space in the nonlinear version of the problem considered here yields large scale dynamo action for a wider range of parameter values.

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